1. Measures of Central tendency

* Central tendency Means average behaviour of a Series or distribution.
* An average represents the characteristics of the data
* There are two Measures of Central tendency
  - i, Mathematical averages
  - ii, Positional averages
* Mathematical averages are :-
  - i, Arthematic Mean
  - ii, Geometric Mean
  - iii, Harmonic Mean
* Positional averages are :-
  - i, Median
  - ii, Mode
* Measures of Central tendency are also Called as first order averages.

Arithmetic Mean:

A.M is the Most Commonly used Measure of Central tendency. It is a Mathematical average. therefore it Can be used for furthere Algebric treatment.

$$\boxed{AM = \overline{X} = \frac{\Sigma fm}{\Sigma f}}$$

Median: It is a positional average. In the Calculation of Median the given data is arranged in accending Order.

$$\boxed{Median = L + \frac{\frac{N}{2} - Cf}{f} \times i}$$

where L = Lower Limit of the Median Class
f = frequency of the " "
Cf = Cumulative frequency of the class above the Median class
i = Size of the Median class.

Mode: It is also a positional average. Mode or Modal Value Means Most frequently occuring item.

$$\boxed{Mode = L + \frac{f_1 - f_0}{2f_1 - f_0 + f_2} \times i}$$

where L = Lower limit of the Modal Class.
f_1 = frequency of the Modal class.
f_0 = " " " Class above the Modal class
f_2 = " " " class below the Modal class
i = Size of the Modal class.

1) Class : 0-10 10-20 20-30 30-40 40-50 50-60 60-70

frequency: 4 8 10 16 11 7 4

Calculate partime deviation & its Co-efficient.

$$\boxed{\frac{Q_3-Q_1}{2}} \qquad \boxed{\frac{Q_3-Q_1}{Q_3+Q_1}}$$

Class	frequency f	cumulative frequency c.f
0-10	4	4
10-20	8	12
20-30	10	22
30-40	16	38
40-50	11	49
50-60	7	56
60-70	4	60

$N = 60$

Q_1 class $= \frac{N}{4}^{th}$ item

$= \frac{60}{4} = 15^{th}$ item $= 20-30$

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times i = 20 + \frac{15-12}{10} \times 10 = \underline{23}$$

Q_3 class $= 3\frac{N}{4}^{th}$ item

$3 \times \frac{60}{4} = 45^{th}$ item $= 40-50$

$$Q_3 = L + \frac{3N/4 - cf}{f} \times i = 40 + \frac{45-38}{11} \times 10 = \underline{46.36}$$

$\therefore$ Partime deviation $= \frac{46.36 - 23}{2}$

$= 11.68$

Co-efficient $= \frac{23.36}{69.36}$

$= \underline{0.336}$

Q2) H.W

Class	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60-69
freq	2	5	8	13	7	5

Calculate partime deviation & its Co-efficient

$Q_1 = 33.25$

$Q_3 = 52.35$

P.D $= 9.55$

Co-eff $= 0.22$

Standard deviation :

Standard deviation is the Most Commonly used Measure of dispersion. it is denoted by a Greek letter "σ" (sigma)

Formulae :

1. individual series $\boxed{\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}}$ Mean

2. Discrete series $\boxed{\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2}}$ Mean

3. Continuous series $\boxed{\sigma = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}}$ Mean

Q1) Calculate S.D for the following data

12, 10, 16, 10, 22, 13, 15, 17, 15, 20

X	X^2
12	144
10	100
16	256
10	100
22	484
13	169
15	225
17	289
15	225
20	400
150	2392 Σx^2

$N = 10$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{2392}{10} - \left(\frac{150}{10}\right)^2}$$

$$= \sqrt{239.2 - 225}$$

$$= \sqrt{14.2}$$

$$= \underline{3.76}$$

f	3	10	20	30	15	12	10

Calculate S.D

$$\sigma = \sqrt{\frac{\Sigma f x^2}{N} - \left(\frac{\Sigma f x}{N}\right)^2}$$

X	X^2	f	fx	fx^2
5	25	3	15	75
15	225	10	150	2250
25	625	20	500	12500
35	1225	30	1050	36750
45	2025	15	675	30375
55	3025	12	660	36300
65	4225	10	650	42250
		N=100	Σfx=3700	Σfx^2=160500

$$= \sqrt{\frac{160500}{100} - \left(\frac{3700}{100}\right)^2}$$

$$= \sqrt{1605 - (37)^2}$$

$$= \sqrt{1605 - 1369}$$

$$= \sqrt{236}$$

$$= 15.36$$

Q3.) Class 0-10 10-20 ...

freq	15	15	23	22	25	10	5	10

Calculate S.D

$$\sigma = \sqrt{\frac{\Sigma f m^2}{N} - \left(\frac{\Sigma f m}{N}\right)^2}$$

Class	Midvalue (M)	M^2	f	f.M	$f.M^2$
0-10	5	25	15	75	375
10-20	15	225	15	225	3375
20-30	25	625	23	575	14375
30-40	35	1225	22	770	26950
40-50	45	2025	25	1125	50625
50-60	55	3025	10	550	30250
60-70	65	4225	5	325	21125
70-80	75	5625	10	750	56250
			N=125	Σfm=4395	Σfm^2=203325

$$\sigma = \sqrt{\frac{203325}{125} - \left(\frac{4395}{125}\right)^2}$$

$$= \sqrt{1626.6 - 1236.225}$$

$$= \sqrt{390.38}$$

$$= 19.75$$

** Co-efficient of Variation (C.V)

Co-efficient of Variation is a relative Measure of Standard deviation. It is used to find the Consistency (or) Variability of the data. C.V is used to Make a Comparitive study B/w two or More Series.

$$C.V = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

$$\boxed{C.V = \frac{\sigma}{\bar{x}} \times 100}$$

* Less C.V = Less Variability = More Consistency
  More C.V = More Variability = Less Consistency.

IMP

Q1) The performance of two cricketers in ten innings is given below.

A:	25	50	45	30	70	42	36	48	34	60
B:	10	70	50	20	95	55	42	60	48	80

find who is better run getter & who is More Consistant player.

Hint (when the words Consistancy or Variability are found in the given problem CV is to be Calculated).

A

X	X^2
25	625
50	2500
45	2025
30	900
70	4900
42	1764
36	1296
48	2304
34	1156
60	3600
$\Sigma x = 440$	$\Sigma x^2 = 21070$

B

X	X^2
10	100
70	4900
50	2500
20	400
95	9025
55	3025
42	1764
60	3600
48	2304
80	6400
530	34,018

$$\boxed{\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}}$$

$\text{Mean} = \frac{\Sigma x}{N} = \frac{440}{10} = 44$

$\sigma = \sqrt{\frac{21070}{10} - \left(\frac{440}{10}\right)^2}$

$= \sqrt{2107 - 1936}$

$= \sqrt{171}$

$= 13.076$

$\text{Mean} = \frac{530}{10} = 53$

$\sigma = \sqrt{\frac{34018}{10} - \left(\frac{530}{10}\right)^2}$

$= \sqrt{3401.8 - 2809}$

$= \sqrt{592.8}$

$= 24.34$

$C.V = \frac{24.34}{53} \times 100$

$= 45.92$

$C.V = \frac{S.D}{Mean} \times 100 \Rightarrow \frac{13.076}{44} \times 100 = 29.71$

interpretation:

1) average of Player A is 44, " " " B is 53 } ∴ Player B is Better Run getter

2 C.V of player A is 29.71, " " B is 45.92 } ∴ player A is More Consistent player

Q) Two ... ten innings find who is Better Run getter & who is More Consistent player.

A	42	17	83	59	72	76	64	45	40	32
B	28	70	31	0	59	108	82	14	3	95

A

x	x^2
42	1764
17	289
83	6889
59	3481
72	5184
76	5776
64	4096
45	2025
40	1600
32	1024
530	32128

Player A is More Consistent

Player B is Better Run getter.

B

x	x^2
28	784
70	4900
31	961
0	0
59	3481
108	11664
82	6724
14	196
3	9
95	9025
490	37,744

Mean $\frac{\Sigma x}{N} = 53$

$\sigma = \sqrt{\frac{32128}{10} - \left(\frac{530}{10}\right)^2}$

$= \sqrt{3212.8 - 2809}$

$= 20.09$

$CV = \frac{20.09}{53} \times 100 = 37.9$

Mean $= \frac{490}{10} = 49$

$\sigma = \sqrt{3774.4 - (49)^2}$

$= \sqrt{1373.4}$

$= 37.09$

$C.V = \frac{37.09}{49} \times 100$

$= 75.63$

Clo

28/10

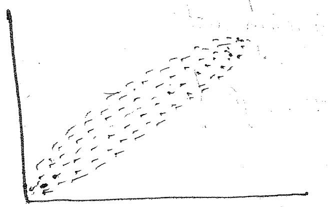
4. CO-Relation

* CoRelation Means the Relationship b/w two inter-related Variables
* CoRelation co-efficient is Caliculated to Measure the extent of association b/w two or More Variables
* The Objectives of Co-Relation are :-
  1. finding the existence of association b/w Variables
  2. Measuring the association.
* CoRelation May be :-
  1. Simple CoRelation (two variables)
  2. Multible CoRelation (three or More Variables)
* CoRelation May also be :-
  1. Positive CoRelation
  2. Negative CoRelation
* When the Variables Move in the Same direction, it is called positive CoRelation.
* When the variables Move in opposite direction, it is called negative Co-Relation.
* Co-Relation Value always lies b/w −1 & +1 (both inclusive)
* Co-Relation Co-efficient is denoted by "r"
* If r is +1 ($r = -1$) it indicates perfect positive co-relation
* If r = −1 it indicates perfect negative co-relation
* If r = 0 it indicates spurious co-relation (no ")

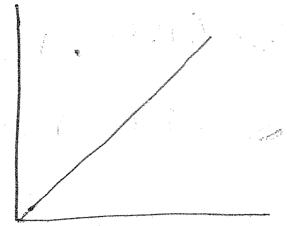
Methods of Co-relation:

1. Scatter diagram Method
2. Karl pearson Method
3. Spearman's Rank Co-relation Method
4. Concurrent deviation Method.

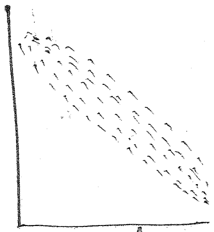
1.) **Scatter diagram Method** :-
it is also called ~~Dot~~ Dot diagram Method
This Method is used Just to know the nature of Corelation b/w the Variables.



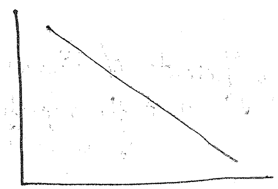
i, Positive Correlation
(r = +ve)



ii, Perfect Positive Correlation
(r = +1)



iii, Negative Correlation
(r = −ve)



iv, Perfect Negative Co-relation
(r = −1)

v, No Correlation
(r = 0)

2) **Karl pearson Method** :-
This Method is used for Quantitative aspects (100% Measurable) like Sales & profit, Cost & Profit, etc.
Under this Method :-

$$\frac{\ldots}{\sqrt{N\Sigma x^2-(\Sigma x)^2}\sqrt{N\Sigma y^2-(\Sigma y)^2}}$$

2) $r = \dfrac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$ $x = X - \bar{X}$, $y = Y - \bar{Y}$

3) The significance of correlation can be tested with the concept of probable Error (P.E)

$$\boxed{P.E = 0.6745 \cdot \frac{1-r^2}{\sqrt{N}}}$$

If $r \geq 6P.E$, r is significant.

If $r \leq 6P.E$, r is not significant.

29/10

Q

X	2	5	8	7	10	15	12	20	25	18
Y	3	7	9	8	12	12	10	18	25	20

Calculate Karl pearson Co-efficient of Correlation. Test the significance of Co-rrelation also

X	X^2	Y	Y^2	XY
2	4	3	9	6
5	25	7	49	35
8	64	9	81	72
7	49	8	64	56
10	100	12	144	120
15	225	12	144	180
12	144	10	100	120
20	400	18	324	360
25	625	25	625	625
18	324	20	400	360
Σx = 122	Σx^2 = 1960	124	1940	1934

$$r = \frac{10 \times 1934 - (122)(124)}{\sqrt{10(1960)-(122)^2} \cdot \sqrt{10(1940)-(124)^2}}$$

$$= \frac{19340 - 15128}{\sqrt{19600-14884} \cdot \sqrt{19400-15376}}$$

$$= \frac{4212}{\sqrt{4716} \cdot \sqrt{4024}} = \frac{4212}{68.67 \times 63.43}$$

$$= \frac{4212}{4355.73}$$

$r = +0.967$

$\boxed{P.E = 0.6745 \times \frac{1-r^2}{\sqrt{N}}}$

$$P.E = 0.6745 \times \frac{1-(0.967)^2}{\sqrt{10}}$$

$= \cancel{7.0318 \times 10^{-3}}$ $\boxed{0.013}$

$6P.E = 6 \times 0.013 = 0.078$

$r = 0.967$

$r > 6PE \therefore r$ is significant

2) 5

X	23	27	24	28	30	31	33	35	36	39
Y	18	22	24	23	25	26	28	29	30	32

Calculate Karl pearson's correlation co-efficient & test the significance.

$r = 0.995$

X	X^2	Y	Y^2	XY
23	529	18	324	414
27	729	22	484	594
29	841	24	576	696
28	784	23	529	644
30	900	25	625	750
31	961	26	676	806
33	1089	28	784	924
35	1225	29	841	1015
36	1296	30	900	1080
39	1521	32	1024	1248
311	9875	257	6763	8171

$\Sigma x = 4$, $\Sigma y = 8$, $\Sigma x^2 = 28$, $\Sigma y^2 = 76$, $\Sigma xy = 46$

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{10 \times 8171 - (311)(257)}{\sqrt{10 \times 9875 - (311)^2}\sqrt{10 \times 6763 - (257)^2}} = \frac{1783}{\sqrt{2029} \cdot \sqrt{1581}}$$

$=$

$\therefore r = +0.995$

$$P.E = 0.6745 \times \frac{1-(0.995)^2}{\sqrt{10}} = 2.1276 \times 10^{-3}$$

$6PE = 0.012$

$r = 0.995$

$r > 6PE$

$\therefore r$ is significant

3) 6 from the given particulars calculate co-efficient of correlation.

	X	Y
No. of items	7	7
Arithmetic Mean	4	8
Sum of the squares of deviations taken from Mean	28	76
Sum of the product of deviations of x & y series from their respective Means	46	

$N = 7$

$\Sigma x^2 = 28$

$\Sigma y^2 = 76$

$\Sigma xy = 46$

Σx^2
Σy^2
Σxy

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$$

$$= \frac{46}{\sqrt{28 \cdot 76}} = \frac{46}{\sqrt{2128}} = \frac{46}{46.130}$$

$= +0.997$

Q Total of Product of deviations of X & Y series = 85
No. of pairs of observations = 10
Total of the deviations of X series = 6
Total of the " of Y series = 13
Total of the squares of deviations of X series = 162
" " " " of Y series = 211

find out the co-efficient of correlation when arbitary ~~Market~~ Mean of X series & Y series are 35 & 30 Respectively

assumed Mean.

$$r = \frac{85}{\sqrt{162 \cdot 211}} = \frac{85}{\sqrt{34182}} = \underline{0.459}$$

	$x-\bar{x}$	$X-A$
X	x	dx
y	y	dy

$\Sigma dx \cdot dy = 85$
$N = 10$
$\Sigma dx = 6$
$\Sigma dy = 13$

$\Sigma dx^2 = 162$
$\Sigma dy^2 = 211$

$$r = \frac{\ldots}{\sqrt{N\sum dx^2-(\sum dx)^2} \cdot \sqrt{N\sum dy^2-(\sum dy)^2}}$$

$$= \frac{10 \times 85 - 78}{\sqrt{1620-36} \cdot \sqrt{2110-169}} = \frac{772}{39.79 \times 44.05}$$

$$= 0.410$$

Spearman's Rank Correlation Method :-

This Method is used for qualitative aspects like honesty, ~~duty~~ beauty, wisdom or Morality etc.

There are 3 Cases in the Calculation of Rank Correlation Co-efficient.

1. When Ranks are given
2. When Ranks are not given
3. For equal Ranks.

Case I When Ranks are given:

correlation

$$\boxed{r = 1 - \frac{6\sum D^2}{N^3 - N}}$$

Q1) The Ranks of the same 15 students in different subjects A & B are given Below

(1, 10), (2, 7), (3, 2), (4, 6), (5, 4), (6, 8), (7, 3) (15, 13) (14, 12) (13, 14), (11, 9), (12, 5), (8, 1), (9, 11), (10, 15)

Use Spearman's formula to find the Co-Relation Coefficient

$$r = 1 - \frac{6\sum D^2}{N^3 - N}$$

$N = 15$

$N = 15$

Rank in A R_1	Rank in B R_2	$D = R_1 - R_2$	D^2
1	10	9	81
2	7	5	25
3	2	1	1
4	6	2	4
5	4	1	1
6	8	2	4
7	3	4	16
8	1	7	49
9	11	2	4
10	15	5	25
11	9	2	4
12	5	7	49
13	14	1	1
14	12	2	4
15	13	2	4
			272

$$r = 1 - \frac{6\,(272)}{15^3 - 15} = 1 - \frac{1632}{3360} = +0.514$$

2) 10 Competitors in a beauty contest are ranked by 3 judges in the following order.

Judge 1 :	1	6	5	10	3	2	4	9	7	8
" 2 :	3	5	8	4	7	10	2	1	6	9
" 3 :	6	4	9	8	1	2	3	10	5	7

Use the Rank correlation co-efficient to determine which pair of Judges has the nearest approach to common ~~Pairs~~ tastes in beauty

J_1 & J_2

Ranks by J_1	Ranks by J_2	$D = J_1 - J_2$	D^2
1	3	2	4
6	5	1	1
5	8	3	9
10	4	6	36
3	7	4	16
2	10	8	64
4	2	2	4
9	1	8	64
7	6	1	1
8	9	1	1
			200

$$r = 1 - \frac{6(200)}{10^3 - 10}$$

$$= 1 - \frac{1200}{990}$$

J_1 & J_2 $= -0.212$

J_2 & J_3

Ranks by J_2	Ranks by J_3	D	D^2
3	6	3	9
5	4	1	1
8	9	1	1
4	8	4	16
7	1	6	36
10	2	8	64
2	3	1	1
1	10	9	81
6	5	1	1
9	7	2	4
			214

$$r = 1 - \frac{6(214)}{990}$$

$= -0.296$

J_3 & J_1

J_3	J_1	D	D_2
6	1	5	25
4	6	2	4
9	5	4	16
8	10	2	4
1	3	2	4
2	2	0	0
3	4	1	1
10	9	1	1
5	7	2	4
7	8	1	1
			60

J_1 & J_2 = -0.212

J_2 & J_3 = -0.296

J_3 & J_1 = $+0.636$

$$r = 1 - \frac{6(60)}{990}$$

$r = +0.636$

Rank Correlation ...

11 Maximum i.e., +0.636 therefore they have the nearest approach

Case II When Ranks are not given.

Q

X	58	43	50	19	26	27	75	34	28	67
Y	23	24	34	16	25	14	30	39	17	29

Calculate Rank Correlation Co-efficient.

Least number, least to highest

X	R_1	Y	R_2	$D = R_1 - R_2$	D^2
58	8	23	4	4	16
43	6	24	5	1	1
50	7	34	9	2	4
19	1	16	2	1	1
26	2	25	6	4	16
27	3	14	1	2	4
75	10	30	8	2	4
34	5	39	10	5	25
28	4	17	3	1	1
67	9	29	7	2	4
					76

$$r = 1 - \frac{6(76)}{990} = 0.539$$

Case III Equal Ranks

12 When there are more than one item with same Ranks a common Rank is given to such items. then Co-rrelation is calculated as follows

$$r = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots\right]}{N^3 - N}$$

Where $m_1, m_2 \ldots$ is no of times an item is repeated

Q

X	15	20	28	12	40	60	20	80
Y	40	30	50	30	20	10	30	60

Calculate Rank Correlation Co-efficient.

X	R_1	Y	R_2	$D = R_1 - R_2$	D^2
15	2	40	6	4	16
* 20	3 3.5	* 30	3 4	0.5	0.25
28	5	50	7	2	4
12	1	* 30	4 4	3	9
40	6	20	2	4	16
60	7	10	1	6	36
* 20	4 3.5	* 30	5 4	0.5	0.25
80	8	60	8	0	0
					81.5

take average

in x series $m_1 = 2$

in y series $m_2 = 3$

Vinas

$$r = 1 - \frac{6\left[81.5 + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(3^3 - 3)\right]}{8^3 - 8}$$

$$= 1 - \frac{6[81.5 + 0.5 + 2]}{512 - 8} = 1 - \frac{504}{504}$$

$= 1 - 1 = 0$

13

Concurrent deviation Method :

Under this Method Magnitude (Value) of change is ignored But direction of change is taken :

$$\boxed{r = \pm\sqrt{\pm\frac{2c-n}{n}}}$$

where c = no. of +ve deviations

n = no. of pairs compared

n = N − 1

where N = no. of observations

For increase = "+"

For decrease = "−"

For no change = 0

Q. Calicculate co-efficient of Concurrent deviation for the following data

x	200	250	290	275	275	300	250	280
y	45	50	60	60	55	62	50	42

X	Dx	Y	Dy	Dx Dy
200		45		
250	+	50	+	+
290	+	60	+	+
275	−	60	0	0
275	0	55	−	0
300	+	62	+	+
250	−	50	−	+
280	+	42	−	−
				C = 4

count only + signs

14

$$r = \pm\sqrt{\pm\frac{2(4)-7}{7}}$$

$$= \pm\sqrt{\pm\frac{1}{7}}$$

$$= 0.377$$

$$= +0.37$$

4/11'15

5. REGRESSION

→ CORRELation analysis is concerned with finding and Measuring the Relationship b/w two or More Variables. Regression analysis is an extension of Correlation analysis.

→ The purpose of Regression is estimating the Value of one Variable from the known Value of the other Variable.

→ If there are two variables, there will be two regression equations.

→ Therefore, Regression is Concerned with formation of equations to ~~estimate~~ Make estimation.

→ for 2Variables X & Y there are two Regression equations

Regression Equation of X on Y
(X- Equation)

$$X - \bar{X} = bxy(Y - \bar{Y})$$

$$\Rightarrow \boxed{X = \bar{X} + bxy(Y - \bar{Y})}$$

Regression equation of Y on X
(Y-Equation)

$$Y - \bar{Y} = byx(X - \bar{X})$$

$$\boxed{Y = \bar{Y} + byx(X - \bar{X})}$$

Vinay

Kumar

10 Regression Co-efficients:

1. bxy, byx are called Regression Co-efficients
2. They are also called slopes of the Regression equations.
3. bxy is called Regression co-efficient of X equation
4. byx is called " " " Y equation
5. $\boxed{bxy = \frac{\Sigma xy}{\Sigma y^2}}$, $x = X - \bar{X}$, $y = Y - \bar{Y}$

$$\boxed{bxy = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma Y^2 - (\Sigma Y)^2}}$$

$$bxy = r \cdot \frac{\sigma_x}{\sigma_y}$$

6. $\boxed{byx = \frac{\Sigma xy}{\Sigma x^2}}$

$$\boxed{byx = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma X^2 - (\Sigma X)^2}}$$

$$byx = r \cdot \frac{\sigma_y}{\sigma_x}$$

7. CoRRelation Coefficient

Vinay Kumar

$$r = \sqrt{bxy \cdot byx}$$

8. both the Regression Co-efficients show the same sign i.e., either positive or negative.
9. If the Regression Co-efficients are "+ve" Correlation coeffic- will be +ve
10. " " " are "-ve" r will be -ve

Q1)

Particulars	X	Y
Arithmetic Mean.	39.5	47.5
Standard deviation	10.8	17.8
Co-relation coefficient	+0.42	

Find

a. Regression Eq of X on y
b. " " of Y on x
c. Value of X for Y = 30
d. Value of Y for X = 50

Σx Σy σx σy r σ

a) $X = \bar{X} + bxy(y - \bar{y})$

$$bxy = r \cdot \frac{\sigma x}{\sigma y} = 0.42 \times \frac{10.8}{17.8}$$

$$= 0.254$$

$$X = 39.5 + 0.254(y - 47.5)$$

$$= 39.5 + 0.254y - 12.065$$

$$\therefore X = 27.435 + 0.254y$$

b. $y = \bar{Y} + byx(x - \bar{x})$

$$=$$

$$byx = r \cdot \frac{\sigma y}{\sigma x}$$

$$= r \cdot 0.42 \times \frac{17.8}{10.8}$$

$$= 0.692$$

$$y = 47.5 + 0.692(x - 39.5)$$

$$= 47.5 + 0.692x - 27.342$$

$$\therefore y = 20.15 + 0.692x$$

14 c) Value of x for y = 30

$$x = 27.435 + 0.254(30)$$

$$\therefore x = 35.055 \underset{appx}{\simeq} 35$$

d) Value of Y for x = 50

$$Y = 20.15 + 0.692(50)$$

$$Y = 54.75 \underset{appx}{\simeq} 55$$

2)

	X	Y
Mean	36	85
Variance	121	64
SD	"	8
Correlation	0.66	

$\sqrt{v \cdot s} = S.D.$

find the following A. Regression equation X on Y
" " Y on X.
Value of x for Y = 60
Value of Y for x = 50

(Hint: $\sqrt{\text{Variance}} = SD$)

a) $X = \bar{X} + bxy(y - \bar{y})$

$bxy = \sqrt{\text{Variance}} = SD$
$= \sqrt{121} = 11$

3)

Particulars	Advt X	Sales Y
Mean	10	90
S.D	3	12
Correlation	0.8	

A $x = 0.2y - 8$
$y = 3.2x + 58$
B 106
C 16

find the

A. Calculate the two Regression Lines
B. find the likely Sales when Advt expenditure is 15 /-
C. What should be Add to expenditure if the Company wants attain a sale target of 120 /-

A) $x = \bar{X} + bxy\,(Y - \bar{Y})$ $bxy = r \cdot \frac{\sigma x}{\sigma y} = 0.8 \cdot \frac{3}{12} = 0.2$

$x = 10 + 0.2(y - 90)$
$= 10 + 0.2y - 1.8$
$x = \cancel{8.2}\ 0.2y - 8$

$Y = \bar{Y} + byx\,(x - \bar{x})$ $byx = 0.8 \times \frac{12}{3} = 3.2$
$= 90 + 3.2(x - 10)$
$= 90 + 3.2x - 32$
$Y = 58 + 3.2x$

B) $Y = 58 + 3.2x$ Advt $= 15$, $x = 15$
$Y = 58 + 3.2(15)$
$y = 106$

C) $X = 0.2y - 8$ Sales target $= 120$, $y = 120$.
$= 0.2(120) - 8$
$= 16$

4 Imp

X :	25	28	35	32	31	36	29	38	34	32
Y :	43	46	49	41	36	32	31	30	33	29

find the following
A. find Regression eq. of X on Y
B. " " " Y on X.
C. Correlation Co-efficient
D. Value of X for Y = 40
E. Value of Y for X = 30

X	X^2	Y	Y^2	XY
25	625	43	1849	1075
28	784	46	2116	1288
35	1225	49	2401	1715
32	1024	41	1681	1312
31	961	36	1296	1116
36	1296	32	1024	1152
29	841	31	961	899
38	1444	30	900	1140
34	1156	33	1089	1122
32	1024	29	841	928
$\Sigma x =$ 320	10380	380	14838	12067
		390	14158	11747

$\bar{X} = \frac{\Sigma X}{N}$

$\bar{Y} = \frac{\Sigma Y}{N}$

Regression Equation of X on Y

$X = \bar{X} + bxy (Y - \bar{Y})$

$\bar{X} = \frac{\Sigma X}{N} = \frac{320}{10} = 32$

$\bar{Y} = \frac{\Sigma Y}{N} = 38$

$bxy = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma Y^2 - (\Sigma Y)^2}$

$= \frac{10(12067) - (320)(380)}{10(14838) - (380)^2}$

$= \frac{-930}{3980} = -0.233$

Then $X = 32 + (-0.2333)(Y - 38)$

$\boxed{X = 40.854 - 0.233Y}$ — 1

Regr. Equation of Y on X :-

$Y = \bar{Y} + byx (X - \bar{X})$

$byx = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma X^2 - (\Sigma X)^2} = \frac{10(12067) - (320)(380)}{10(10380) - (320)^2}$

$= -0.664$

$\therefore \boxed{Y = 59.248 - 0.664X}$ — 2

C. Correlation coefficient

$r = \sqrt{bxy \cdot byx}$

$= \sqrt{(-0.233)(-0.664)}$

$= 0.393$

$\therefore r = +0.393$ — 3

D $Y = 40 \quad X = ?$

$X = 40.854 - 0.233Y$

$= 31.534$

$X = 30 \quad Y = ?$

$Y = 59.248 - 0.664X$

$= 39.328$

5) / 2

X	10	10	18	25	28	33	34	39	42	41
Y	13	22	22	19	35	27	33	40	42	47

Find the following:
a, regression eq of x on y
b, " " y on x
c Co-relation Co-efficient
d Value of x for y = 50.
e, Value of y for x = 50

Ans: a $x = 0.956y - 0.68$
b. $y = 0.817x + 7.124$
c $r = 0.884$
d $x = 47.12$
$y = 47.974$

X	X^2	Y	Y^2	XY
10	100	13	169	130
10	100	22	484	220
18	324	22	484	396
25	625	19	361	475
28	784	35	1225	980
33	1089	27	729	891
34	1156	33	1089	1122
39	1521	40	1600	1560
42	1764	42	1764	1764
41	1681	47	2209	1927
280	9144	300	10,114	9465

Regression eq of x on y

$$X = \bar{x} + b_{xy}(y - \bar{y})$$

$$= 28 + 0.956(y - 30)$$

$$X = 28 + 0.956y - 28.68$$

$$x = 0.956y - 0.68$$

$\bar{x} = \frac{\Sigma x}{N} = \frac{280}{10} = 28$

$\bar{y} = \frac{\Sigma y}{N} = \frac{300}{10} = 30$

$$b_{xy} = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{N\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{10(9465) - (280)(300)}{10(10114) - (300)^2}$$

$$= \frac{10,650}{11,140} = 0.956$$

2) Regression of y on x

$$Y = \bar{y} + b_{yx}(x - \bar{x})$$

$$Y = 30 + 0.817(x - 28)$$

$$y = 0.817x + 7.124$$

$$b_{yx} = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{94650 - (280)(300)}{91440 - (280)^2}$$

$$= \frac{10650}{13,040} = 0.81$$

c) $r = \sqrt{b_{xy} \cdot b_{yx}}$

$$= \sqrt{(0.956)(0.817)}$$

$$r = 0.884$$

d, $X = 0.956(50) - 0.68$
$= 47.12$

e, $Y = 0.817(50) + 7.124$
$= 47.974$

Least squares Method:

→ Under this Method a trend eq. is formed the trend eq. is used to predict the values of the variable for the Comming years.

→ The Trend eq is $Y_c = a + bX$

→ Here Y_c = trend value

a = Intercept (mean of the given value)

b = Slope of the eq.

X = deviation from the Middle period

→ The Value of a & b can be found as follows.

$$a = \frac{\Sigma Y}{N} \qquad b = \frac{\Sigma XY}{\Sigma X^2}$$

Q

Year:	2007	2008	2009	2010	2011	2012	2013
Sales:	10	12	13	16	15	18	21

Fit a straight line trend eq and obtain trend values for all the years and also estimate sales for 2015

Q3

Year	Sales y	Deviation from Middle period (x)	x^2	xy	Trend Value $Y_c = 15+1.68x$
2007	10	−3	9	−30	9.96
2008	12	−2	4	−24	11.64
2009	13	−1	1	−13	13.32
$\boxed{2010}$	16	0	0	0	0
2011	15	1	1	15	16.68
2012	18	2	4	36	18.36
2013	21	3	9	63	20.04
$N=7$	$\Sigma y = 105$	$\Sigma x = 0$	$\Sigma x^2 = 28$	Σxy 47	

2011

$$\boxed{Y_c = a + bx}$$

$a = \frac{\Sigma y}{N} = \frac{105}{7} = 15$

$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{47}{28} = 1.68$

$$\therefore \boxed{Y_c = 15 + 1.68x}$$

Trend value.

2007 $\quad Y_c = 15 + 1.68(-3) = 9.96$

2008 $\quad Y_c = 15 + 1.68(-2) = 11.64$

Estimated sales for 2015

for 2013 $x = 3$

$\therefore$ for 2015 $x = 5$

$Y_c = 15 + 1.68(5)$

$= \boxed{23.4}$

Q2

Year	2008	2009	2010	2011	2012	2013
profits	12	10	14	11	13	15

Form a Trend eq and obtain Trend Value for all the 6 years. estimate profit for 2017

Year	profits y	Deviation from Middle period	x^2	xy	Y_c
2008	12	−2.5	6.25	−30	11
2009	10	−1.5	2.25	−15	11.6
2010	14	−0.5	0.25	−7	12.2
2011	11	0.5	0.25	5.5	12.8
2012	13	1.5	2.25	19.5	13.4
2013	15	2.5	6.25	37.5	14
$N=6$	$\Sigma y = 75$	$\Sigma x = 0$	$\Sigma x^2 = 17.5$	$\Sigma xy = 10.5$	

2010.5

$a = \frac{\Sigma y}{N} = 12$

$b = \frac{\Sigma xy}{\Sigma x^2} = 0.6$

$$\boxed{Y_c = 12.5 + 0.6x}$$

Y_c for 2017 = 16.4

estimated sales for

2014 $\quad x = 3.5 \quad Y_c = 14.6$

⋮

2017 $\quad x = 6.5 \quad Y_c = 16.4$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11}(b_{11}) + a_{12}(b_{21}) & a_{11}(b_{12}) + a_{12}(b_{22}) \\ a_{21}(b_{11}) + a_{22}(b_{21}) & a_{21}(b_{12}) + a_{22}(b_{22}) \end{bmatrix}$$

$$\begin{bmatrix} 2(-3)+(-1)(2)+3(-1) & 2(2)-5-6 & -2-2+3 \\ 9+2(0)+0 & -6+10+0 & 3+4+0 \\ 15+0-1 & 10+5+2 & 5+2-1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -7 & -1 \\ 9 & 14 & 7 \\ -16 & 17 & -4 \end{bmatrix}$$

$\boxed{AB \neq BA}$

2)* If $A = \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 0 \\ 5 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 & -1 \\ 0 & 5 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

find AB & BA show that $AB \neq BA$.

$$AB = \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 0 \\ 5 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 2 & -1 \\ 0 & 5 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6-0+3 & -4-5-6 & -2-2+3 \\ +9+0+0 & -6+10 & +3+4 \\ -15-1 & 10+5+2 & -5+2-1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -7 & -1 \\ +9 & 4 & 7 \\ -16 & 17 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 2 & -1 \\ 0 & 5 & 2 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 0 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -6-6-5 & 3+4-1 & -9+ \\ -15+10 & 10+2 & -2 \\ 2+6+5 & -1-4+1 & 3-1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -17 & 6 & -8 \\ -5 & 12 & -2 \\ 13 & -4 & 2 \end{bmatrix}$$

$\therefore AB \neq BA$ (AD - BC)

3) $A = \begin{bmatrix} 8 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

find AB & BA & show that $AB \neq BA$

$BA \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 3 \\ 12 & 4 & 1 \end{bmatrix}$ $AB \begin{pmatrix} 11 & 4 & 12 \\ 1 & 1 & 10 \\ -1 & 5 & -4 \end{pmatrix}$

$$AB = \begin{pmatrix} 8+3 & -2+6 & 16-4 \\ 2-1 & 3-2 & 4+6 \\ -3+2 & 1+4 & -6+2 \end{pmatrix} = \begin{pmatrix} 11 & 4 & 12 \\ 1 & 1 & 10 \\ -1 & 5 & -4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 8-6 & -2+2 & 3+4 \\ 2-6 & 3+2 & -1+4 \\ +8+4 & -2+6 & 3-2 \end{pmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 3 \\ 12 & 4 & 1 \end{bmatrix}$$

$\therefore AB \neq BA$

Determinant:

determinant is the Value of a square Matrix which is used to solve Linear eq. in the form of Matrices. → det. A $|A|$

4) find the determinant of the following Matrix.

$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ B =

$$|A| = 1\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} - 2\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} + (-2)\begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix}$$

$= 3 + 2 - 4$

$= 1$

$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} = -2$

$1 \, |-1 - 1| +$

$= +2$

$C = \begin{pmatrix} 1 & 2 & 5 \\ 7 & 3 & 4 \\ 5 & -1 & -6 \end{pmatrix} =$

$1|-18-4| - 2|-42-20| + 5|-7-15|$

$-22 \quad +124 \quad +40$

$B = \begin{pmatrix} 0 & 0 & 1 \\ 4 & 2 & 1 \\ 9 & -3 & 1 \end{pmatrix} = 31$

$1|-12-18|$

$+30$

$D = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 6 & 1 & 1 \end{pmatrix} = 3$

Inverse of a Matrix :

Inverse of Matrix $A = A^{-1}$

$$\boxed{A^{-1} = \frac{Adj \cdot A}{|A|}}$$

Adj·A = Transpose of Co-factor Matrix A

Q If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \\ 4 & 6 & 5 \end{bmatrix}$ find A^{-1}

$|A| = 1\,|15-18| - 2\,|10-12| + 3\,|12-12|$

$= -3 + 4 + 0$

$= \underline{\underline{+1}}$

Cofactor of $1 = +\begin{vmatrix} 3 & 3 \\ 6 & 5 \end{vmatrix} = -3$

$2 = -\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = +2$

$3 = +\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$

$2 = -\begin{vmatrix} 2 & 3 \\ 6 & 5 \end{vmatrix} = 8$

$3 = +\begin{vmatrix} 1 & 3 \\ 4 & 5 \end{vmatrix} = -7$

$3 = -\begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 2$

$4 = +\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} = -3$

$6 = -\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 3$

$5 = +\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

Cofactor Matrix $A = \begin{bmatrix} -3 & 2 & 0 \\ 8 & -7 & 2 \\ -3 & 3 & -1 \end{bmatrix}$

$\therefore$ Adj A $= A^{T} = \begin{bmatrix} -3 & 8 & -3 \\ 2 & -7 & 3 \\ 0 & 2 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \cdot$ Adj A

$= \frac{1}{1}\begin{bmatrix} -3 & 8 & -3 \\ 2 & -7 & 3 \\ 0 & 2 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -3 & 8 & -3 \\ 2 & -7 & 3 \\ 0 & 2 & -1 \end{bmatrix}$

2) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ find A^{-1} Ans $\begin{pmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{pmatrix}$

$|A| = 0(2-3) - 1(1-9) + 2(1-6)$

$= 0 + 8 - 10$

$= \underline{\underline{-2}}$

Cofactor
$$\begin{pmatrix} -1 & +8 & -5 \\ +1 & -6 & +3 \\ -1 & +2 & -1 \end{pmatrix}$$

$$Adj\,|A| = A^{-1} = \begin{vmatrix} -1 & 1 & -1 \\ +8 & -6 & 2 \\ -5 & 3 & -1 \end{vmatrix}$$

$$A^{-1} = \frac{Adj(A)}{|A|}$$

$$= \begin{vmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & 1 \\ 5/2 & -3/2 & 1/2 \end{vmatrix}$$

28 **Solution of Linear equations (CRAMER'S RULE)**

Linear equations of 3 Variable under CRAMER'S RULE x, y, z, Can be found as follows.

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$$

Q. Solve the following eq. by Cramer's Rule.

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

$$\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

Sol. Let

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(1-1) - 1(-1-2) + 1(1+2)$$
$$= 3 + 3 = 6$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 6(1-1) - 1(-2-1) + 1(2+1)$$
$$= 3 + 3 = 6$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(-2-1) - 6(-1-2) + 1(1-4)$$
$$= -3 + 18 - 3 = 12$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 1(-1-2) - 1(1-4) + 6(1+2)$$
$$= -3 + 3 + 18 = 18$$

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{6}{6} = 1$$
$$y = \frac{\Delta_2}{\Delta} = \frac{12}{6} = 2$$
$$z = \frac{\Delta_3}{\Delta} = \frac{18}{6} = 3$$

19/11/14

Application of Calculus.

The Concept of Caliculas is widely used in decesion Making. In the Cases of Maximization of Profit, Minimization of Cost, elasticity of demand & supply and other Managerial problems. Caliculas is very very useful.

Rules:

1) $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$ eg $= \frac{d}{dx}(x^3) = 3 \cdot x^{3-1} = 3 \cdot x^2$

$\frac{d}{dx}(x^{-5}) = -5x^{-6}$

2) $\frac{d}{dx}(x) = 1$

3) $\frac{d}{dx}(c) = 0$ Constant

4) Revenu = Price x quantity

5) Profit = Revenu − Cost

6) Marginal Revenu = derivative of Revenue $= \frac{d}{dx}(R)$

7) Marginal Cost = derivative of cost $= \frac{d}{dx}(c)$

8) Marginal profit = derivative of profit $= \frac{d}{dx}(P)$

9) at Break even point (BEP), Revenue = Cost

10) Profit is Maximum at MP = 0.
ie., MR = MC

11) Cost is Minimum at MR = MC (or) MC = 0

1) $y = 5x^5 + 7x^4 - 8x^3 + 4x^2 + 3x + 9$
differentiate the above fn. w.r.to x

$y = 5x^5 + 7x^4 - 8x^3 + 4x^2 + 3x + 9$ (write in detail)

$\frac{dy}{dx} = 25x^4 + 28x^3 - 24x^2 + 8x + 3$

2) $y = 6x^6 - 7x^5 + 8x^4 + 9x^3 - 2x^2 - 5x - \frac{6}{x^4} + \frac{5}{x^3} - \frac{2}{x^2} + \frac{7}{x} - 12$

$\frac{d}{dx}(6x^6)$ $6\frac{d}{dx}x^6$ $-\frac{d}{dx}(-7x^5)$

differentiate the above fn. w.r.to x. $\boxed{\frac{1}{a^m} = a^{-m}}$

$\frac{dy}{dx} = \frac{d}{dx}\left(6x^6 - 7x^5 + 8x^4 + 9x^3 - 2x^2 - 5x - 6x^{-4} + 5x^{-3} - 2x^{-2} + 7x^{-1} - 12\right.$

$\frac{dy}{dx} = 36x^5 - 35x^4 + 32x^3 + 27x^2 - 4x - 5 + 24x^{-5} - 15x^{-4} + 4x^{-3} - 7x^{-2}$

$\frac{dy}{dx} = 36x^5 - 35x^4 + 32x^3 + 27x^2 - 4x - 5 + \frac{24}{x^5} - \frac{15}{x^4} + \frac{4}{x^3} - \frac{7}{x^2}$

(3)

$6x^2+6x-36=0$

$x^2+x-6=0$

$x^2+3x-2x-6=0$

$x(x+3)-2(x+3)$

$x=\underline{2 \text{ or } -3}$

(4) 31) The Cost of Producing a Certain article is given by $C = 10+\frac{80}{x}+5x^2$ find the Minimum Value of x

Sol

given.

$C = 10+\frac{80}{x}+5x^2$

$\frac{dc}{dx} = -\frac{80}{x^2} + 10x$

$\therefore \frac{dc}{dx} = 10x - \frac{80}{x^2}$

$10x - \frac{80}{x^2} = 0$

$10x = \frac{80}{x^2}$

$10x^3 = 80$

$x^3 = 8$

$x = 2$

Marginal Cost = derivative of Cost.

Cost is Minimum at MR = MC (or) MC = 0

$\therefore$ x will be Minimum at $x=2$

5) Find Maxima & Minima of the f^n.

***

$y = 2x^3+3x^2-36x+10$

given

$\frac{dy}{dx} = 6x^2+6x-36$

$6x^2+6x-36=0$

$6x^2+18x-12x-36=0$

$6x(x+3)-12(x+3)=0$

$6x-12=0$

$x=2$

$x+3=0$

$x=-3$

$6\times6\times6$

$6\times3\times2\times1$

18×12

~~Maxima = 2~~

~~Minimum = -3~~

Pages-8,14

$x = -3$ given Maxima as the function

$$y = 2x^3 + 3x^2 - 36x + 10$$

$$= 2(-3)^3 + 3(-3)^2 - 36(-3) + 10$$

$$= 91$$

$x = 2$ given Minima of the function.

$$y = 2x^3 + 3x^2 - 36x + 10$$

$$= 2(2)^3 + 3(2)^2 - 36(2) + 10$$

$$= 16 + 12 - 72 + 10$$

$$= -34$$

$x = -3$ Max

$x = 2$ Min.

EOQ = Economic Ordering Quantity.

The Quantity of which total Purchase of the Material is Minimum

Hint for EOQ, derivative of Cost is Made equal to zero.

Given:

$$T = 10000 + \frac{250000}{Q} + \frac{Q}{16}$$

$$\frac{dT}{dQ} = \frac{1}{16} - \frac{250000}{Q^2}$$

For EOQ, $\frac{dT}{dQ} = 0$

$$\frac{1}{16} - \frac{250000}{Q^2} = 0$$

$$Q^2 = 250000 \times 16$$

$$Q = 2{,}000 \text{ units} \quad \text{EOQ.}$$

Cost at 2,500 units.

$$T = 10000 + \frac{25{,}0000}{Q} + \frac{Q}{16}$$

$$= 10000 + \frac{250{,}000}{2{,}500} + \frac{2{,}500}{16}$$

$$T = 10{,}256.25$$

7) $C = -300x - 10x^2 + \frac{1}{3}x^3$ where C = Cost

32 x = output

find a, output at which MC is Minimum
b, output at which MAC is Minimum
c, o/p at which MC = AC

Hint: $AC = \text{Cost}/\text{Quantity}$

given

$$C = -300x - 10x^2 + \frac{1}{3}x^3$$

A, $MC = \frac{dc}{dx} = -300 - 20x + x^2$

MC is Minimum at $\frac{dc}{dx} = 0$

$$x^2 - 20x - 300 = 0$$

$$x^2 - 30x + 10x - 300 = 0$$

$$x(x-30) + 10(x-30) = 0$$

$$x = 30 \quad \text{or} \quad x = -10$$

3 x 100
3 x 25 x 4
3 x 5 x 5 x 4
15
3 x 5 x 5 x 2 x 2
30

$\therefore$ MC is Minimum at $x = 30$ units.

$\because$ (It is non-negative)

b, Marginal Average Cost

$$AC = \frac{\text{Cost}}{\text{Quantity}} = \frac{-300x - 10x^2 + \frac{1}{3}x^3}{x}$$

$$= -300 - 10x + \frac{1}{3}x^2$$

$$MAC = \frac{d(AC)}{dx}$$

$$= \frac{d}{dx}\left(-300 - 10x + \frac{1}{3}x^2\right)$$

$$= -10 + \frac{2}{3}x.$$

33 MAC is Minimum at $\frac{d}{dx}(AC) = 0$

$$-10 + \frac{2}{3}x = 0$$

$$\frac{2}{3}x = 10$$

$$x = \frac{30}{2} = 15 \text{ units}$$

MAC is Minimum at 15 Units.

c) $MC = -300 - 20x + x^2$

$AC = -300 - 10x + \frac{1}{3}x^2$

MC = AC

$$\cancel{-300} - 20x + x^2 = \cancel{-300} - 10x + \frac{1}{3}x^2$$

$$x^2 - \frac{1}{3}x^2 = 10x$$

$$3x^2 - x^2 = 30x$$

$$2x^2 = 30x$$

$$x = 15$$

3 | 3, 9, 27
3 | 1, 3, 9
1, 1, 3

8, find Maxima & Minima of $y = x^3 - 2x^2 + x + 6$

$$\frac{dy}{dx} = 3x^2 - 4x + 1$$

$\frac{dy}{dx} = 0$

$$3x^2 - 3x - x + 1$$

$$3x(x-1) - 1(x-1)$$

$$(3x-1)(x-1)$$

$$x = 1/3 \quad \text{or} \quad x = 1$$

at $x = 1$: $1 - 2 + 1 + 6$

$= 6$

at $x = 1/3$

$$\frac{1}{27} - \frac{2}{9} + \frac{1}{3} + 6$$

$$\frac{1}{27}$$

*

26/11

Probability and Expected Value

* Probability means possibility of happening or not happening an event.
* The event is also called an experiment.
* every experiment produces some out come.
* The out come may be favourable or unfavourable.
* favourable outcome is called sucess
* un " " " failure.

→ Probability of sucess is denoted by $P(E)$ (or) p

→ Probability of failure is denoted by $P(\bar{E})$ or q

$$P(E) = \frac{\text{no. of favourable cases.}}{\text{Total no. of cases.}}$$

$$\boxed{P(E) = \frac{\text{Favourable Cases}}{\text{Exhaustive Cases.}} = \frac{FC}{EC}}$$

$$\boxed{P(\bar{E}) = 1 - P(E)}$$

* Probability value always lies b/w 0 & 1 (Both inclusive)

Coins :- for 1 Coin, $EC = 2^1 = 2$ (H,T)

for 2 Coins, $EC = 2^2 = 4$ (HH, HT, TH, TT)

for 3 Coins, $EC = 2^3 = 8$ (HHH, TTT, HHT, HTH, THH, TTH, THT, HTT)

for n Coin, $EC = 2^n$

DICE

For 1 Dice, $EC = 6^1 = 6$

2 Dice $EC = 6^2 = 36$

3 Dice $EC = 6^3 = 216$

n Dice $EC = 6^n$

Pack

52 Card
- Red (26)
  - Hearts (13) ♡
  - Diamonds (13) ◇
- Black (26)
  - Spades (13) ♤
  - Clubs (13) ♧

No of suits = 4

No of cards in each suit = 13

No. of face cards = 4 (J, K, Q, A)

No of number cards = 9 (2 to 10)

No of Court Cards = 3 (J, Q, K)

Knave Card = 1 (J)

Balls:

9) 3 White, 4 Red, 5 green balls.

a) No of white ball = 3

No of Red balls = 4

No of G.B = 5

∴ Total no. of balls = 12.

a white ball

$P(WB) = \frac{3}{12}$

10) No. of Red Balls = 3

" of G. Balls = 4

" " Y. Balls = 5

Total no. of balls = 12

2 balls are drawn

$\therefore EC = {}^{12}C_2 = \frac{12 \times 11}{2} = 66$

a, Red balls. $P(R \cdot B) = \frac{3C_2}{66} = \frac{3}{66} = \frac{1}{22}$

b, Green balls $P(G.B) = \frac{4C_2}{66} = \frac{6}{66} = \frac{1}{11}$

c, ~~5~~ Yellow $P(Y.B) = \frac{5C_2}{66} = \frac{10}{66}$

d, $P(R.B \text{ or } G.B) = \frac{7C_2}{66} = \frac{21}{66}$

e, $P(G.B \text{ or } Y.B) = \frac{9C_2}{66} = \frac{36}{66}$

f, $P(R \text{ or } Y \text{ balls}) \quad \frac{8C_2}{66} = \frac{28}{66}$

g) = Any two different colour balls.

$P(RB \,\&\, G.B)$ or $P(RB \,\&\, YB)$ or $P(GB \,\&\, YB)$

$= \left(\frac{3C_1 \times 4C_1}{12C_2}\right) + \left(\frac{3C_1 \times 5C_1}{12C_2}\right) + \left(\frac{4C_1 \times 5C_1}{12C_2}\right)$

$= \frac{47}{66}$

16) No. of White balls = 3

" Black " = 4

" Red " = 5

12 balls.

Probability of taking a white ball in the first attempt is $3/12$

it is not replaced.

Probability of taking a black ball in the second attempt = $4/11$

" " Red ball in the 3^{rd} attempt = $5/10$

∴ Required probability = $\frac{3}{12} \times \frac{4}{11} \times \frac{5}{10} = \frac{1}{22} = 0.045$

15) c 5 White Balls and 3 black balls 2 balls are drawn one after the other with out Replacement

c) Balls of different Colour

We Require 2 different colour balls

there are 2 Cases White Ball & Black ball (or) Black ball & White ball.

Case 1 : (W.B & B.B)

Probability of taking a white ball in the first attempt

$P(W.B) = 5/8$ it is not Replaced.

probability of taking a Black ball in the second attempt

$P(B.B) = 3/7$

$\therefore P(W.B \,\&\, B.B) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

Case 2 : (B.B & W.B)

Probability of taking a Black ball in the first attempt

$P(B.B) = \frac{3}{8}$ it is not Replaced

probability of taking a white ball in the second attempt

$P(W.B) = \frac{5}{7}$

$\therefore P(B.B \,\&\, W.B) = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

$\therefore$ Require probability $= \frac{15}{56} + \frac{15}{56} = \frac{30}{56}$

14) With Replacement

No. of R.B = 8

No of G.B = 5

Total no. of balls = 13

3 balls are drawn in each attempt

We Require 2RB & 1GB in first attempt & 1RB & 2GB in 2nd attempt.

Probability of drawing 2RB & 1GB in first attempt

$= \frac{{}^8C_2 \times {}^5C_1}{{}^{13}C_3}$

The drawn balls are Replaced.

Probability of drawing 1 Red.B and 2 GB in the Second attempt

$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3}$

$\therefore$ Total probability $= \frac{{}^8C_2 \times {}^5C_1}{{}^{13}C_3} \times \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3}$

With out Replacement :

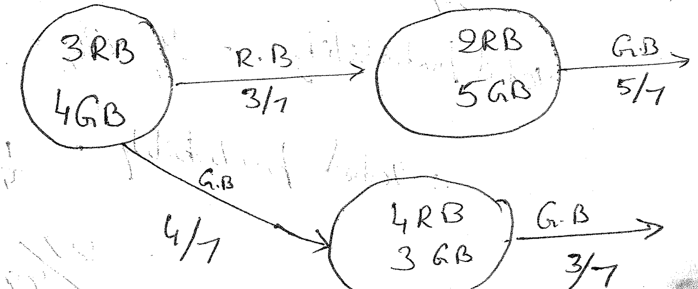
Probability of taking 2R.B & 1 G.B in the first attempt $= \frac{{}^8C_2 \times {}^5C_1}{{}^{13}C_3}$

These 3 balls are not Replaced.

now probability of taking 1 R.B & 2 G.B in the second attempt $= \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$

Total probability $= \frac{{}^8C_2 \times {}^5C_1}{{}^{13}C_3} \times \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$

11)



$\frac{15}{49}$

(+)

$\frac{12}{49}$

$= \frac{27}{49}$

one ball is drawn in the first attempt & one green ball in the second attempt it is possible in two ways.

Way-1 (RB & G.B)

No. of Red Balls = 3
No of Green balls = 4 } total = 7

Probability of drawing a Red Ball in the first attempt = $3/7$.
This Red ball is separated & in its place a Green ball is put into the Box. Now the box Contains
2 R.B (3 − 1) & 5 G.B (4+1)

Now probability of drawing a Green ball = $5/7$

Total probability = $\frac{3}{7} \times \frac{5}{7} = \frac{15}{49}$

Way-2 (G.B & G.B)

$P(G.B) = 4/7$

Box Contains 4 R.B (3+1) & 3 G.B (4−1)

in 2nd case

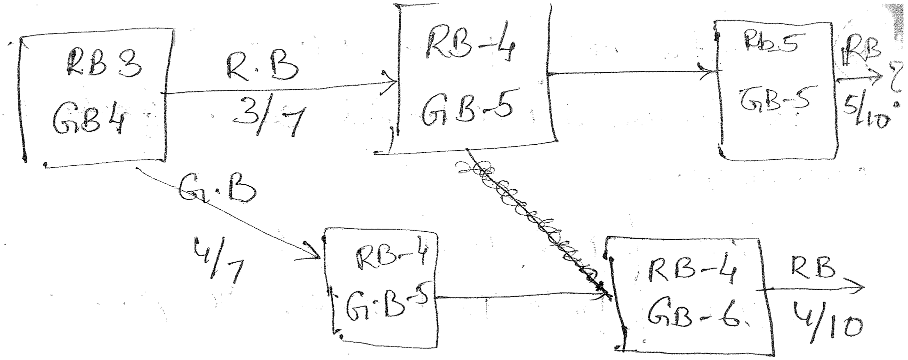
$P(G.B) = 3/7$

Total probability $= \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$

∴ Total probability $= \frac{15}{49} + \frac{12}{49}$

$= 27/49$

12)



$\frac{3}{7} \times \frac{5}{10} = \frac{15}{70}$
(+)
$\frac{4}{7} \times \frac{4}{10} = \frac{16}{70}$ } $31/70$

First box Contains 3 RB & 4 GB
Second box " 4 RB & 5 GB

one ball is drawn from the first box and it is put in the second box it can be done in two ways.

Way 1 (Taking Red ball from the 1st box and RB from the 2nd box)

P (RB from the 1st box) $\frac{3}{7}$

it is put into the 2nd box
Now the 2nd box Contain 5 RB (4+1) & 5 GB

P(R.B from the 2nd box) = $\frac{5}{10}$

Way 2 (Taking G Ball from the 1st box and RB from the 2nd box)

P (G.B from the 1st box) = $4/7$

it is put into the 2nd box

Now 2nd box Contain 4 RB & 6 GB
P (R.B from the 2nd box) = $4/10$

∴ total probability
$(\frac{3}{7} \times \frac{5}{10}) + (\frac{4}{7} \times \frac{4}{10})$
$= 31/70$

19) Total no. of persons $(EC) = 5$

We require a male member & a ~~female~~ members over 30 years

No. of male members $= 3$

No. of female members over 30 years $= 1$

$\therefore$ $FC = (3+1) = 4$

$\therefore$ Require probability $= \frac{FC}{EC} = \frac{4}{5}$

20) Probability of solving the problem by A is

$P(A) = \frac{1}{2} \Rightarrow P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$

" " by B is $P(B) = \frac{1}{3} \Rightarrow P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

" " by C is $P(C) = \frac{1}{4} \Rightarrow P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$

" " by D is $P(D) = \frac{1}{5} \Rightarrow P(\bar{D}) = 1 - \frac{1}{5} = \frac{4}{5}$

" " by E is $P(E) = \frac{1}{6} \Rightarrow P(\bar{E}) = 1 - \frac{1}{6} = \frac{5}{6}$

Probability of solving the problem by atleast one person is

$1 - P(\text{Not solving by any one})$

$1 - \left[P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}) \cdot P(\bar{D}) \cdot P(\bar{E})\right]$

$= \frac{5}{6}$

21) $P(\text{Atleast one}) = P(A \text{ or } B) = P(A+B) = P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Let A accountancy

Let B = economics

$P(A) = 3/5$

$P(A \cap B) = 2/5$ Passing both accountancy & economics

$P(A \cup B) = \frac{4}{5}$ passing atleast one subject.

$P(B) = ?$

$\frac{4}{5} = \frac{3}{5} + x - \frac{2}{5}$

$x = \frac{4}{5} - \frac{3}{5} + \frac{2}{5}$

$x = \frac{3}{5}$

$P(B) = \frac{3}{5}$

$1 - \frac{4}{9}$

$5/9$

22 $P(A) = \frac{2}{3}$ $P(\bar{B}) = \frac{4}{9}$ A physics

$P(B) = 5/9$ B chemistry

$P(A \cup B) = \frac{4}{5}$

$P(A \cap B) = ?$

$3 \mid 38.5$

$\quad 13.5$

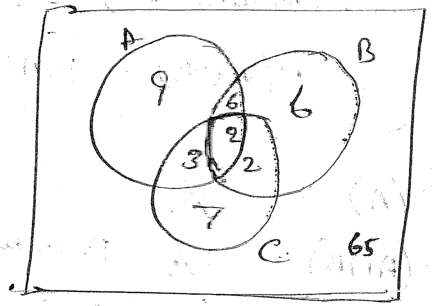
$P(A \cap B) = \frac{2}{3} + \frac{5}{9} - \frac{4}{5}$

$\frac{30 + 25 - 36}{45}$

$55 - 36 = 19$

$P(A \cap B) = 19/45$

23)



Probability of Reading A = $P(A) = 20\% = \frac{20}{100} = 0.2$

" " " B = $P(B) = 16\% = \frac{16}{100} = 0.16$

" " " C = $P(C) = 14\% = \frac{14}{100} = 0.14$

Probability of reading both A and B = $P(A \cap B) = \frac{8}{100} = 0.08$

" " " " A and C = $P(A \cap C) = \frac{5}{100} = 0.05$

" " " " B and C = $P(B \cap C) = \frac{4}{100} = 0.04$

" " " all the three = $P(A \cap B \cap C) = \frac{2}{100} = 0.02$

probability of reading atleast one news paper is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= 0.2 + 0.16 + 0.14 - 0.08 - 0.05 - 0.04 + 0.02$$

$$= 0.35$$

$\therefore$ Probability of not reading any one of the three news papers $= 1 - 0.35$

$= 0.65$

$= 65\%$

24) a) No. of days in a non-leap year = 365

No. of weeks in a year = 52

i.e., $52 \times 7 = 364$ days.

One day is Left the day May be (Sun, Mon, tue, wed, thu, Fri, Sat)

$\therefore$ EC = 7

we require Sunday to get 53 Sundays.

$\therefore$ FC = 1

Then Required probability $= \frac{FC}{EC} = \frac{1}{7}$

b) No. of days in a Leap year = 366

No. of weeks in a year = 52

i.e., $52 \times 7 = 364$ days

2 days Left the days May be

Sun, Mon / Mon, tue / tue, wed / wed, thu / thu, Fri / Fri, Sat / Sat, Sun

$\therefore$ EC = 7

we require Sunday to get 53 Sundays

$\therefore$ FC = 2 (Sun, Mon / Sat, Sun)

The required probability $= \frac{FC}{EC} = \frac{2}{7}$

27) 1st group Contains 3 boys & 1 girl

2nd group Contains 2 boys & 2 girls

3rd group Contains 1 boy & 3 girls

We need (2 boys and 1 girl) are to be selected

it can be done in 3 ways

Way 1 : Boy from 1st group + Boy from 2nd group + Girl from 3rd group

$\Rightarrow$ B B G

Way 2

$\Rightarrow$ B G B

Way 3 $\Rightarrow$ G B B.

Boys	Girls
3	+1
2	+2
1	+3

Probability for Way 1 $= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} = 18/64$

$P(\text{Way}2) = \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} = 6/64$

$P(\text{Way}3) = \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = 2/64$

$\therefore$ Over all probability of selecting 2 boys and 1 girl is

$$\frac{18}{64} + \frac{6}{64} + \frac{2}{64} = \frac{26}{64}$$

28) A speaks truth in 60% cases.
i.e., He tells lies in 40% cases.

$\therefore$ $P(A) = 60\% = 0.6$

$P(\bar{A}) = 40\% = 0.4$

B speaks truth in 70% cases
i.e, he tells lies in 30% cases

$\therefore$ $P(B) = 70\% = 0.7$

$P(\bar{B}) = 0.3$

They are to be in contradiction

i.e, if A tells truth B has to tell lies
if A tells lies B has to tell truth.

$\therefore$ Require Probability $(0.6 \times 0.3) + (0.7 \times 0.4)$

$(0.18) + (0.28)$

$= 0.46$

$= 46\%$

31) No. of boys $= 6$
No. of girls $= 4$.
Total no. of children $= 10$.
3 children are selected at random.

$\therefore$ $EC = {}^{10}C_3 \quad \frac{10 \times 9^3 \times 8^4}{3 \times 2} = 120$

a) No girl i.e, all boys.

$P(3\text{ boys}) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{20}{120} = \frac{1}{6} = 0.16$

b) only one girl. i.e one girl & 2 boys.

$= \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{4 \times 15}{120} = 0.5$

c) one particular girl : 2 boys + 1 Girl

$\frac{{}^6C_2 \times {}^1C_1}{{}^{10}C_3} = 0.125$

e) More girls.

1 boy + 2 girls
0 boys + 3 girls.

$$\frac{({}^6c_1 \times {}^4c_2) + {}^4c_3}{{}^{10}c_3} = \frac{36+4}{120} = \frac{40}{120} = \frac{1}{3}$$

38

Probability of selection of A $= 1/7$

" " " of B $= 1/5$

i.e., probability of not selecting A $= 6/7$

" " " B $= 4/5$

i, P(Both of them will be selected) $= \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$

$P(A) \cdot P(B)$

ii, P(only one will be selected) $= \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{6}{7} \times \frac{1}{5}\right) = \frac{10}{35}$

$P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$

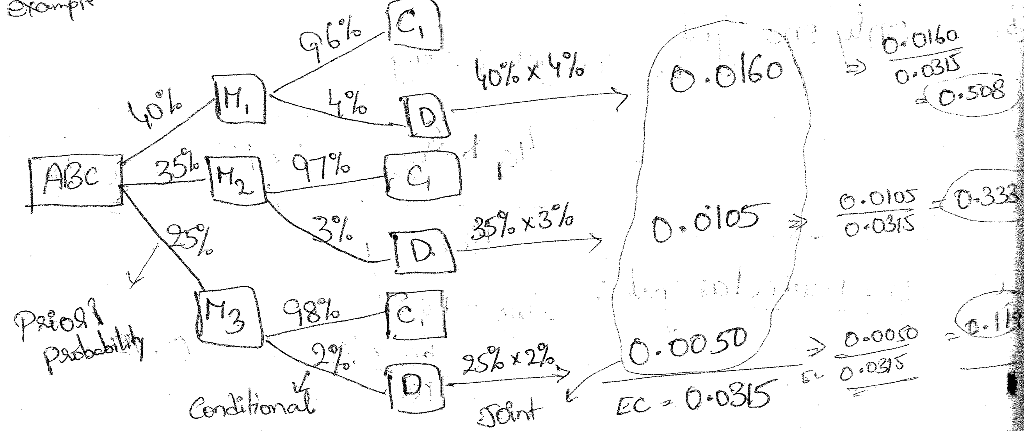
iii, P(none will be selected) $= \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$

$P(\bar{A}) \times P(\bar{B})$

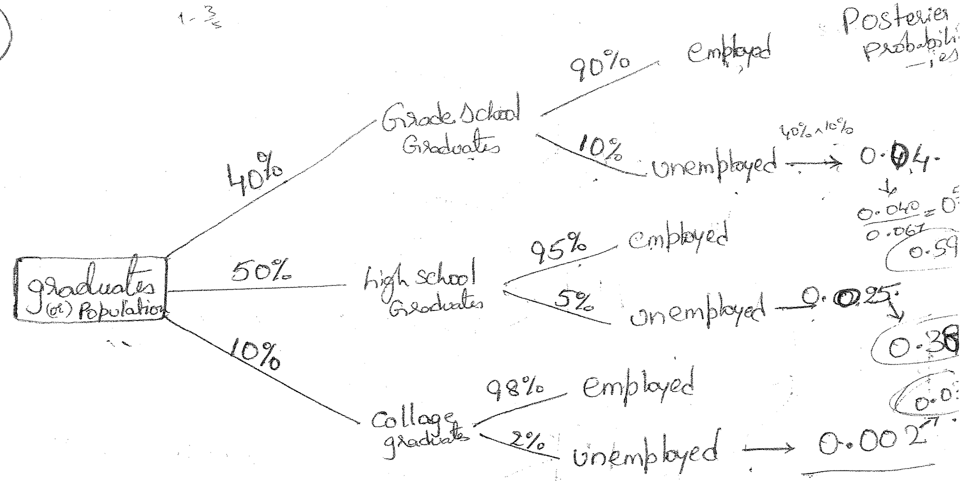
Baye's theorem :

$\frac{40}{100} \times \frac{4}{100}$ $\frac{16}{10000}$

example



39)

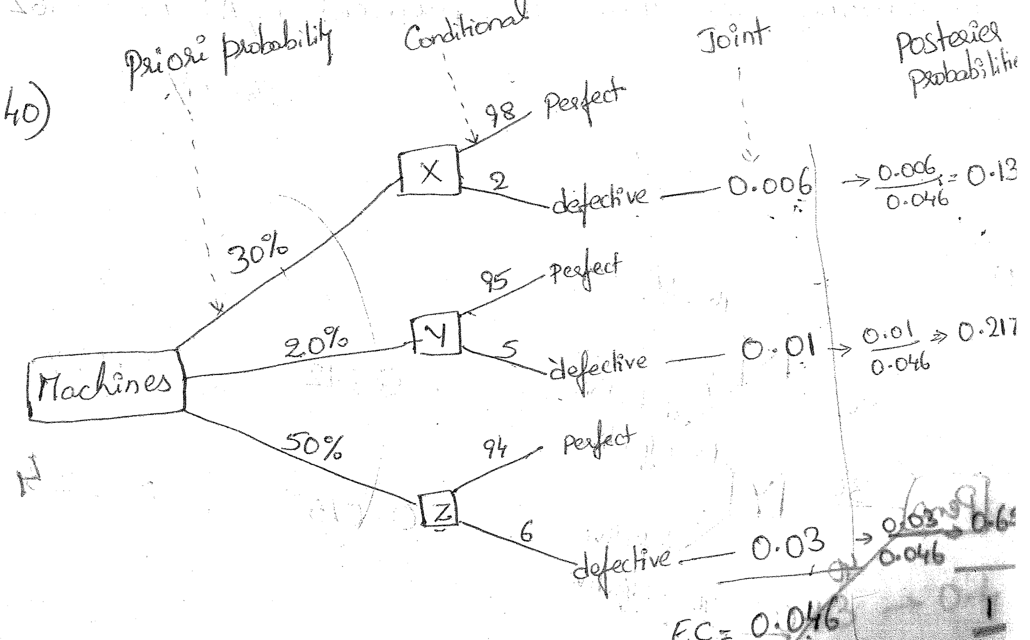


P(selecting an unemployed person from college graduate) EC = 0.067

∴ probability that he is a college graduate is

0.03 = 3%

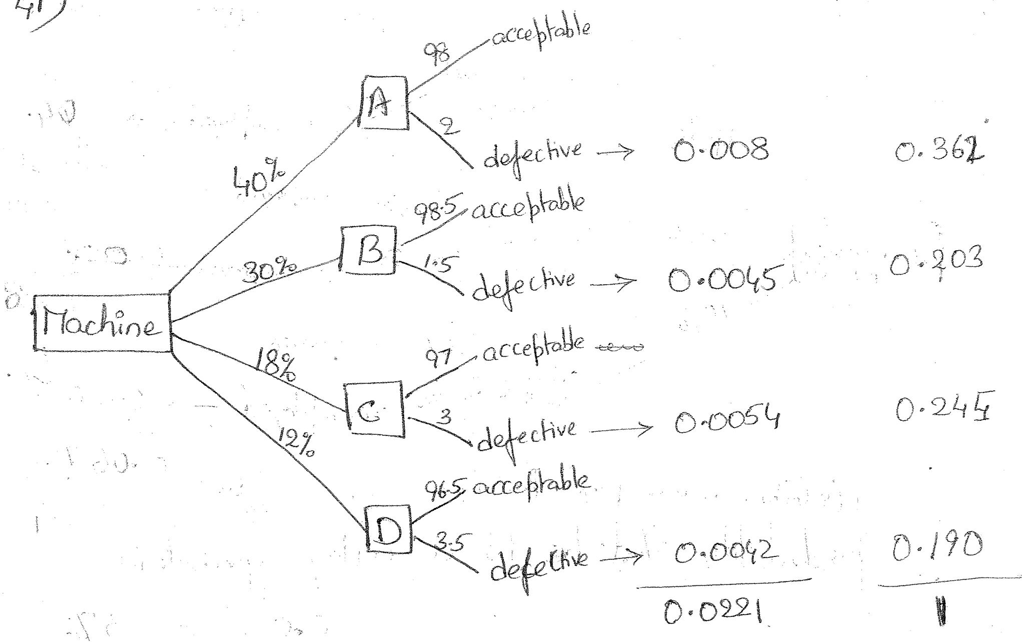
40)



∴ P(selecting a defective item which was produced by machine X) = 0.13

= 13%

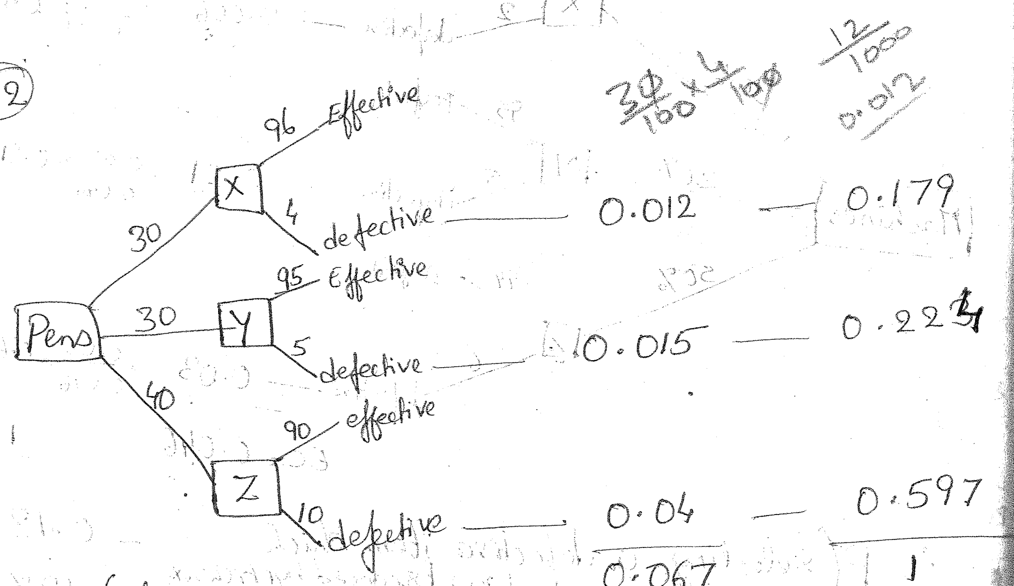
41)



P(Selecting defective item produced by A) is 0.362

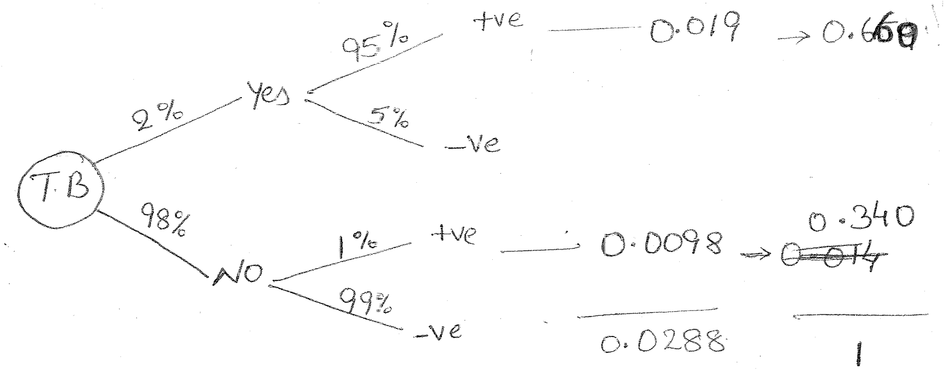
= 36.2%

42)



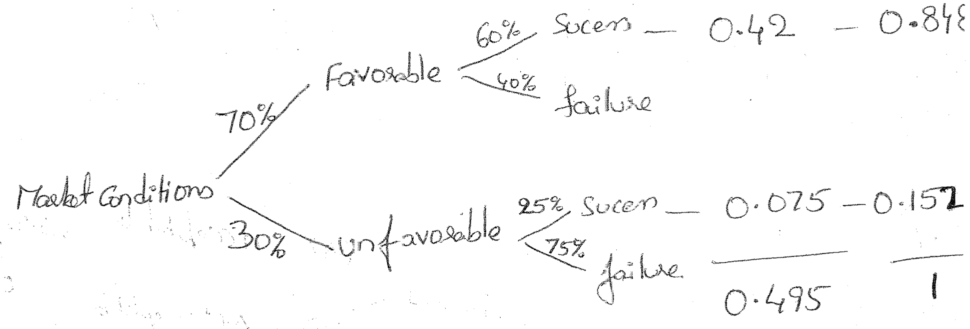
P(Selecting defective item produced by Machine Z) = 59.7%

43)



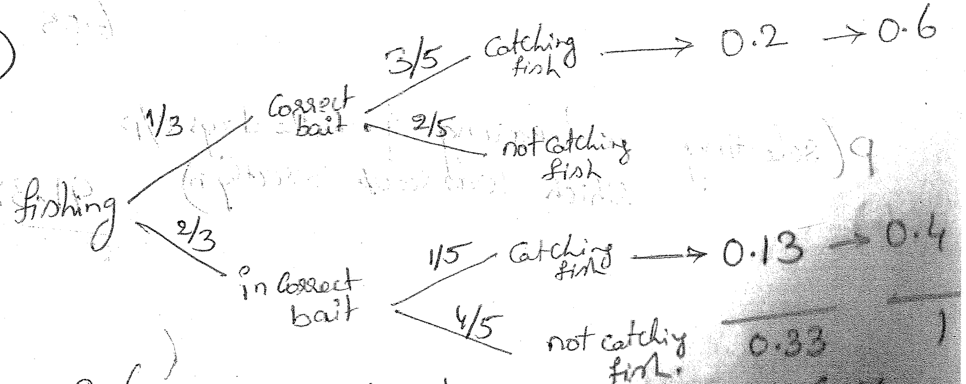
P(person not affected by +B) = 66%

44)



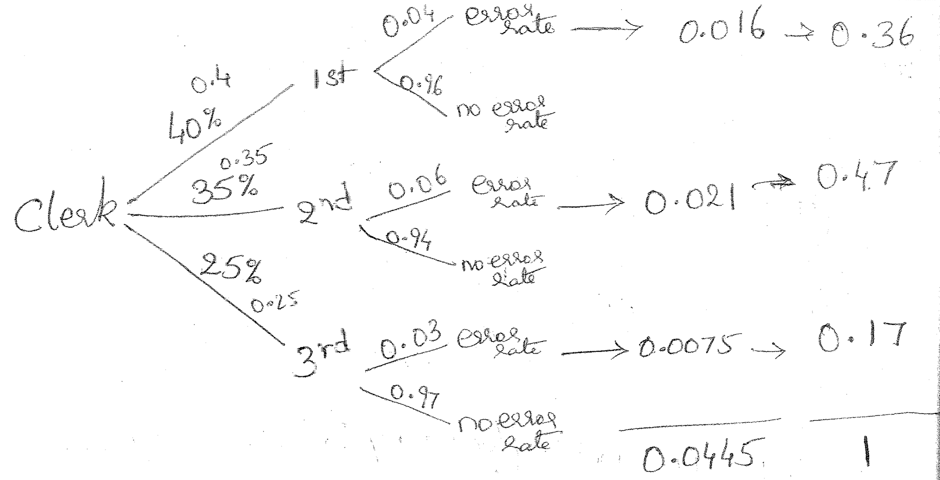
P(Market Conditions favourable) = 84.8%

45)



P(Using Correct bait for Catching fish) = 60%

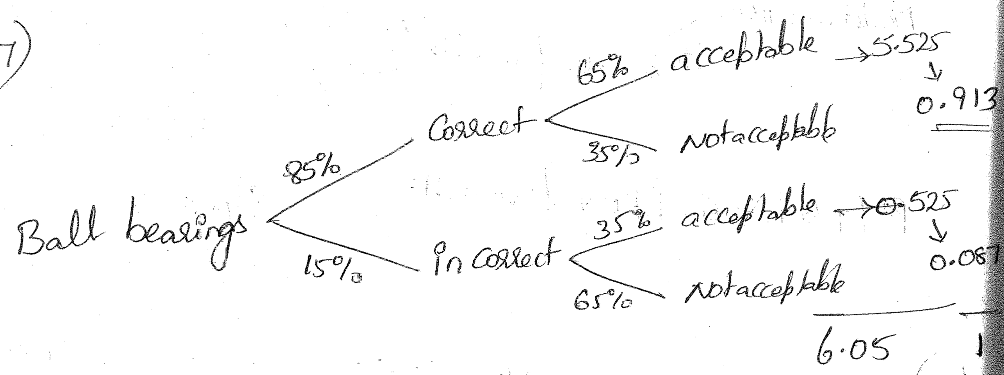
46)



P(error from first clerk) = 36%

P(error from second clerk) = 17%

47)



P(selecting a bearing from the days o/p which was setup correctly is) 91.3%

9. Theoretical distributions:

→ It is also called probability distributions or expected frequency distributions.

→ There are 3 important Theoretical distributions

1. binomial distribution
2. poisson "
3. Normal "

→ Binomial & poisson distributions are discrete probability distribution.

→ Normal distribution is a continious probability distribution

Binomial distribution

→ It is a bi-parametric distribution (n and p are the two parameters)

→ B.D is used where the probabilities of sucess & failure (P & Q) are more or less equal.

→ Under B.D, Probability is calculated by using the following rule

$$\boxed{P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}}$$

Where

n = no. of trials

r = expected no. of sucess

p = probability of sucess

q = $1-p$

frequency = $N \times P(r)$

when $p = q = 1/2$,

$$\boxed{P(r) = {}^nC_r\, p^n}$$

4) $P(r) = {}^nC_r\, p^r q^{n-r}$

Probability of suffering from the disease is

$p = 20\% = 0.2$

not suffering $q = 80\% = 0.8$

$n = 10$ no of workers selected for testig.

$r = 2$

a) $P(r) = {}^{10}C_2 \cdot (0.2)^2 (0.8)^8$

$= 45 \times 0.04 \times 0.1677$

$P(2) = \cancel{0.3019}\ 0.3006$

b) not more than 2 workers suffer from the disease

i.e $r \leq 2$

$P(r \leq 2) = P(0) + P(1) + P(2)$

$P(0) = {}^{10}C_0 \cdot (0.2)^0 (0.8)^{10}$

$= 1 \cdot 1 \times 0.107$

$= 0.107$

$P(1) = {}^{10}C_1 \cdot (0.2)^1 (0.8)^9$

$= 10 \times 0.2 \times 0.134$

$= 0.268$

Required probability

$P(r \leq 2) = 0.107 + 0.268 + 0.302$

$= 0.677$

9). Probability of having boy = Probability of having Girl.

$P(\text{Boy}) = P(\text{Girl})$

$\therefore\ p = q = 1/2$

No. of families $N = 800$

No. of children in each family $(n) = 4$

a) 2 boys and 2 girls.

$r = 2$

$P(r) = {}^nC_r \cdot p^n$

$= {}^4C_2 \cdot (0.5)^4$

$6 \cdot (0.0625)$

$= 0.375$

No. of families having 2 boys and 2 girls.

$\Rightarrow N \times P(2) = 800 \times 0.375 = 300$

b) at least one boy

Boy	Girl
1	+ 3
2	+ 2
3	+ 1
4	+ 0

~~$P(1) = {}^4C_1 \cdot (0.5)^1$~~

$P(\text{At least one boy}) = 1 - P(\text{No boy})$

$r = 0$

$P(0) = {}^4C_0 \cdot (0.5)^4$

$= 0.0625$

$P(\text{At least one boy}) = 1 - 0.0625 \Rightarrow 0.9375$

∴ No. of families having at least one boy is
$800 \times 0.9375 = \underline{\underline{750}}$

c) <u>No. girl</u> i.e., all are boys.

$r = 4$

$P(4) = {}^{4}C_{4} \cdot (0.5)^4$

$1 \cdot (0.0625)$

$= 0.0625$

∴ No. of families having no girl $= 800 \times 0.0625$
$= \underline{50}$

d) <u>at the Most two girls.</u>

2B+2G / 3B+1G / 4B+0G

$\cancel{P(0) + P(1) + P(2)}$

$\cancel{0.0625 + 0.25 + 0.375} =$

$P(2) = 0.375$

$P(3) = {}^{4}C_{3} (0.5)^4 = 0.25$

$P(4) = {}^{4}C_{4} (0.5)^4 = 0.0625$

Req. Probability $= P(2) + P(3) + P(4) = 0.6875$

∴ No. of families having at the Most two girls
$= 800 \times 0.6875$
$= 550$

5) ∴ $N = 256$ no. of times the coins tossed

$n = 8$ no. of coins tossed

∴ $P = q = 1/2$ it is the case of a coin

<u>Calculation of expected frequencies</u>

No. of heads r	$P(r) = {}^{n}C_{r} \cdot p^n$	Expected frequency $= N \times P(r)$
0	$P(0) = {}^{8}C_{0} \cdot (0.5)^8 = 3.906 \times 10^{-3}$	$256 \times 3.906 \times 10^{-3} = 1$
1	$P(1) = {}^{8}C_{1} \cdot (\frac{1}{2})^8 = 8/256$	$256 \times \frac{8}{256} = \underline{\underline{8}}$
2	$P(2) = {}^{8}C_{2} \cdot (\frac{1}{2})^8 = 28/256$	$\frac{256 \times 28}{256} = 28$
3	$P(3) = {}^{8}C_{3} \cdot (\frac{1}{2})^8 = 56/256$	$= 56$
4	$P(4) = {}^{8}C_{4} (\frac{1}{2})^8 = 70/256$	$= 70$
5	$P(5) = {}^{8}C_{5} (\frac{1}{2})^8 = 56/256$	$= 56$
6	$P(6) = {}^{8}C_{6} (\frac{1}{2})^8 = 28/256$	$= 28$
7	$P(7) = {}^{8}C_{7} (\frac{1}{2})^8 = 8/256$	$= 8$
8	$P(8) = {}^{8}C_{8} (\frac{1}{2})^8 = 1/256$	$= 1$

* Mean $= np = 8 \times \frac{1}{2} = 4$
* SD $= \sqrt{npq} = \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = 1.414$

9032366658

8) 59

$p = 0.4$
$q = 0.6$
$n = 5$

a). none
P(none is a graduate).

$r = 0$

$P(0) = {}^5C_0 \cdot (0.4)^0 \cdot (0.6)^5$

$1 \times 1 \times 0.077$

$P(0) = 0.077$

b, one P(one is a graduate)

$r = 1$

$P(1) = {}^5C_1 \cdot (0.4)^1 \cdot (0.6)^4$

$= 5 \times 0.4 \times 0.129$

$= 0.259$

c, at least one will be graduate

$1 - P(0)$

$1 - 0.077$

0.923

10) eight coins are thrown simultaneously

a) at least 6 heads. $n = 8$

$p = q = \frac{1}{2}$

$P(r) = {}^nC_r \cdot p^n$

$P(6) = {}^8C_6 \cdot (\frac{1}{2})^8$

$= 28 \times \frac{1}{256} \Rightarrow 0.109$

$P(7) = {}^8C_7 \cdot (\frac{1}{2})^8 = 8/256 \Rightarrow 0.031$

$P(8) = {}^8C_8 \cdot (\frac{1}{2})^8 = 3.906 \times 10^{-3}$

$P(6) + P(7) + P(8)$

$= 0.1439$

b, no heads

$r = 0$

$P(0) = {}^8C_0 \cdot (\frac{1}{2})^8$

$= 1/256$

c, all heads

$r = 8$

$P(8) = {}^8C_8 \cdot (\frac{1}{2})^8 = 1/256$

Poisson distribution

Poisson distribution is used for Measuring rare events such as printing Mistakes, accidents, defective items, deaths by rare diseases, No. of goals Scored in a football Match etc... Poisson distribution is a unique parametric distribution (Mean (m) is the only one parameter.)

→ under poisson distribution probability is

$$\boxed{P(r) = \frac{e^{-m} \cdot m^r}{r!}}$$

Where $e = 2.7183$
m = mean = np
r = expected no. of sucess

1) Calculation of Mean.

No. of Mistakes (x)	No. of Pages f (N)	fx
0	211	0
1	90	90
2	19	38
3	5	15
4	0	0
$\Sigma x = 10$	$\Sigma f = 325$ (N)	$\Sigma fx = 143$

Mean $m = \frac{\Sigma fx}{N} = \frac{143}{325} = 0.44$

$\frac{\Sigma fx}{\Sigma f}$

Calculation of expected frequencies

$P(r) = \frac{e^{-m} \cdot m^r}{r!} = \frac{e^{-0.44} \cdot (0.44)^r}{r!}$

No. of Mistakes (r)	$P(r)$	expected frequen $N \times P(r)$
0	$P(0) = \frac{0.6443 \cdot (0.44)^0}{0!} = 0.6443$	209.39
1	$P(1) = 0.6443 \times 0.44 = 0.283$	325×0.283 92.134
2	$P(2) = \frac{0.6443 \times (0.44)^2}{2} = 0.062$	20.269
3	$P(3) = \frac{0.6443 \times (0.44)^3}{6} = 9.147 \times 10^{-3}$	2.972
4	$P(4) = \frac{0.6443 \times (0.44)^4}{24} = 1.006 \times 10^{-3}$	0.327
		325.09 aphy

9) No. of bottles in each box = 500

$\therefore n = 500$

Probability of defective bottles = 0.1 % = P

$\therefore$ mean $m = n.P$

$= 500 \times \frac{0.1}{100} = 0.5$

a) No defective:
$r = 0$

$P(0) = \frac{0.6065 \times (0.5)^0}{0!} = 0.6065$

$\therefore$ No. of boxes with no defective $= 0.6065 \times 100$
$= 60.65$
$\simeq$ 61 boxes.

b) Atleast two defective:

$P(r \geq 2) = 1 - P(r < 2)$
$1 - [P(0) + P(1)]$

$P(1) = \frac{0.6065 \times 0.5}{1} = 0.30325$

~~At~~ Then Required probability

$= 1 - (0.6065 + 0.30325)$

$= 1 - 0.90975$

$= 0.09025$

$\therefore$ No. of boxes with atleast two defectives

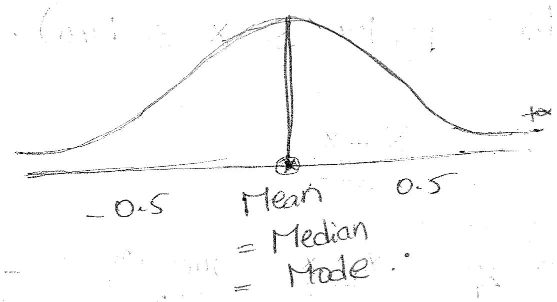
$= 100 \times 0.09025$

$= 9.025$

$\simeq$ 9 boxes

Normal distributions:

→ N.D is a Continious probability distribution to find probability under Normal distribution a symmetrical curve is used. It is a bell shaped curve.



Q For a Normally distributed data, Mean is 120 & S.D is 20 then find the area b/w.

a) 120 & 150
b) 100 & 140
c) More than 160
d) Less than 80

(Note: To find probability in normal distribution, normal probability table is used. For this purpos a standard normal varia. is used it is denoted by 'Z'

$$\boxed{Z = \frac{X - \bar{X}}{\sigma}}$$

Sol:

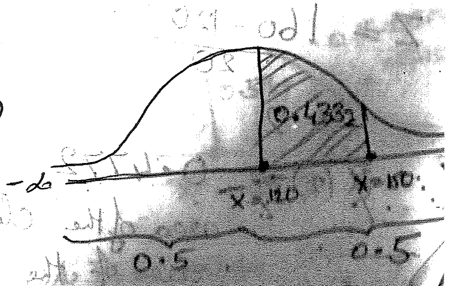
a) $\bar{X} = 120$

$\sigma = 20$

$P(120 \leq X \leq 150) = ?$

$Z = \frac{150-120}{20}$

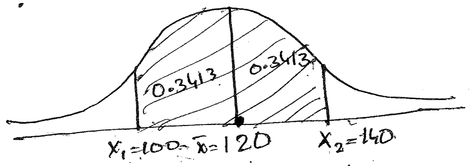
$= 1.50$



from table, area for $Z = 1.50$ is 0.4332

$\therefore P(120 \le X \le 150) = 0.4332$

b) $P(100 \le X \le 140) = ?$



$Z = \frac{X-\bar{X}}{\sigma}$

* $Z_1 = \frac{X_1-\bar{X}}{\sigma} = \frac{100-120}{20} = -1$

$\phi(1) = 0.3413$

* $Z_2 = \frac{X_2-\bar{X}}{\sigma} = \frac{140-120}{20} = 1$

$\phi(1) = 0.3413$

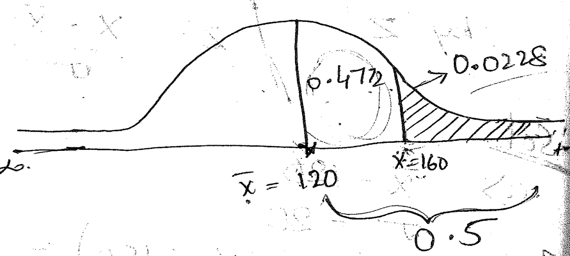
from the table, area for $X_1 \& \bar{X}$, $Z_1 = 1 \Rightarrow 0.3413$ &

$X_2 \& \bar{X}$, $Z_2 = 1 \Rightarrow 0.3413$.

Total area b/w $X_1 \& X_2 = 0.6826$.

$\therefore P(100 \le X \le 140) = 0.6826$

c) $P(X \ge 160) = ?$



$Z = \frac{160-120}{20}$

$= 2$

$\phi(2) = 0.4772$

area of the closed path $= 0.4772$

$\therefore$ area of the remaining path $= 0.5 - 0.4772$

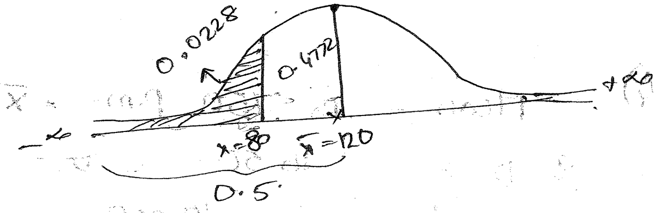
$= 0.0228$

The area to the Right hand side of $x = \ldots$

$\therefore P(X \ge 160) = 0.228$

d) < 80.

$P(X \le 80)$



$Z = \frac{80-120}{20}$

$= -2$

$\phi(2) = 0.4772$

$\therefore$ are to the left hand side of $X = 0.5 - 0.4772$

$= 0.0228$

$\therefore P(X \le 80) = 0.0228$

1) Mean $= \bar{X} = 68.22$

Variance $= 10.8$

$\therefore SD = \sigma = \sqrt{10.8} = 3.28$

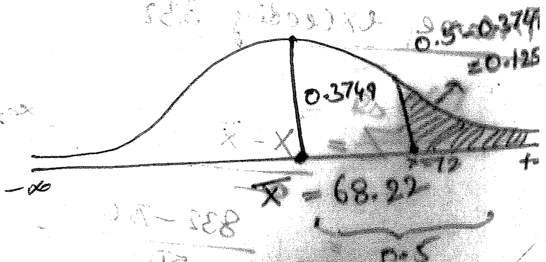
No. of Soldiers $= 1000$

$Z = \frac{X-\bar{X}}{\sigma}$

$= \frac{72-68.22}{3.28}$

$= 1.15$

from table $\phi(1.15) = 0.3749$



$\therefore$ area b/w $\bar{X} \& X = 0.3749$

i.e., area to the right side of X

$\therefore P(\text{Soldiers above 6 feet}) = 0.1251$

$\therefore$ No. of soldiers whose height over 6 feet $= 1000 \times 0.1251$

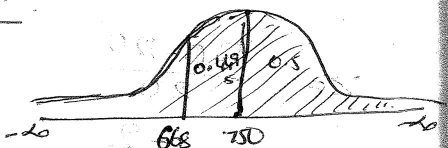
$= 125.1$

$\simeq 125$ soldiers

4) Mean $=$ Rs 750 p.m. $= \bar{x}$

S.D $=$ Rs 50. $= \sigma$

No. of persons $=$ 10,000.



income exceeding Rs 668:

$$Z = \frac{x-\bar{x}}{\sigma} = \frac{668-750}{50}$$

$= -1.64$

from tables $\phi(1.64) = 0.4495$

area ~~b/w~~ to the Right side of x = ~~total~~ income exceeding ~~Rs 668 is~~

$= 0.4495 + 0.5$

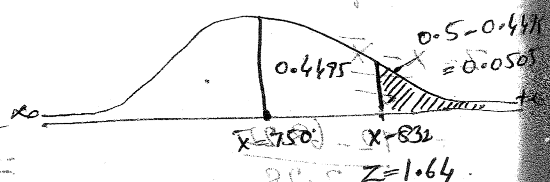
$= 0.9495$

$\simeq 0.95$

$P(\text{people having income exceeding 668 Rs}) = 95\%$ income exceeding Rs 668

income exceeding 832

$$Z = \frac{x-\bar{x}}{\sigma} = \frac{832-750}{50} = 1.64$$



from Tables $\phi(1.64) = 0.4495$

area to the right hand side of x $= 0.5 - 0.4495$

$= 0.0505$

$\simeq 0.05$

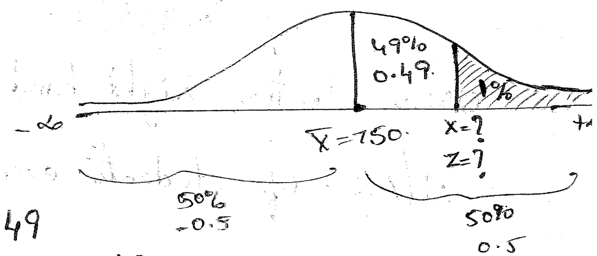
$\therefore P(\text{people having income More than 832 Rs}) = 0.05$

$= 5\%$

Lowest Income of the Richest 100:

% of the Richest 100 persons $= \frac{100}{10000} \times 100$

$= 1\%$



area b/w $\bar{x}$ & $x = 0.49$

from tables, for the area 0.49

$Z = +2.33$

We know

$$Z = \frac{x-\bar{x}}{\sigma}$$

$$2.33 = \frac{x-750}{50} \Rightarrow x = (50 \times 2.33) + 750$$

$= 866.5$

~ 867

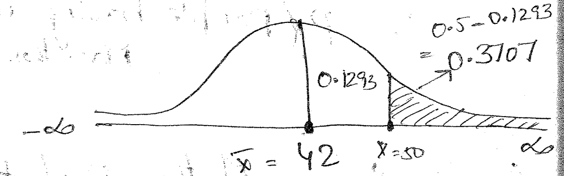
$\therefore$ Lowest income of the Richest 100 $=$ Rs 867

7) No. of students = 1000

average Marks = $\bar{X}$ = 42

S.D. = σ = 24

a) More than 50 Marks:



$$Z = \frac{x - \bar{x}}{\sigma}$$

$$= \frac{50 - 42}{24} = \frac{8}{24} = 0.333$$

$$\phi(0.333) = 0.1293$$

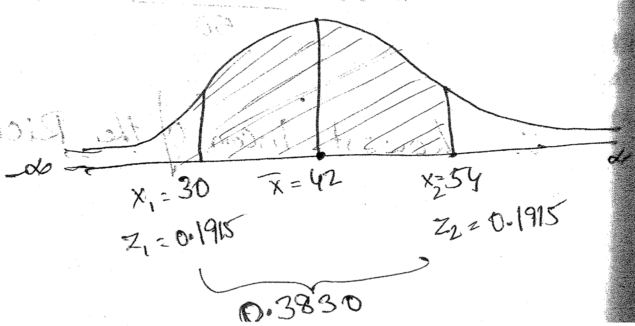
area to the right hand side of x = 0.3707

$\therefore$ P(No. of students exceeding 50 Marks) = 1000 × 0.3707

= 370.7

~ 371

b) No. of Students lying b/w 30 and 54



$$Z_1 = \frac{30 - 42}{24} = 0.5$$

$$\phi(0.5) = 0.1915$$

$\therefore$ area b/w X_1 and $\bar{X}$ = 0.1915

$$Z_2 = \frac{X_2 - \bar{X}}{\sigma} = \frac{54 - 42}{24} = \frac{12}{24} = 0.5$$

$$\phi(0.5) = 0.1915$$

$\therefore$ area b/w X_2 and $\bar{X}$ = 0.1915

$\therefore$ P (students lying b/w 30 and 54) is $Z_1 + Z_2$

= 0.1915 + 0.1915

= 0.3830.

$\therefore$ total area b/w X_1 & X_2 = 0.3830.

then P(30 ≤ x ≤ 54) = 0.3830 × 1000 = 383 students

c) Value of Score exceeds by the top 100 Students.

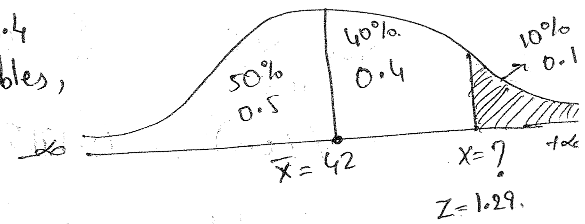
Lowest Marks of top 100 students:

% of top 100 Students in a total of 1000 Students

= 10%

area b/w $\bar{X}$ & X = 0.4

for this area, from tables,



Z = 1.29

$$Z = \frac{X - \bar{X}}{\sigma}$$

$$1.29 = \frac{X - 42}{24}$$

X = (1.29) 24 + 42

= 72.96

~ 73

$\therefore$ the lowest marks of top 100 students

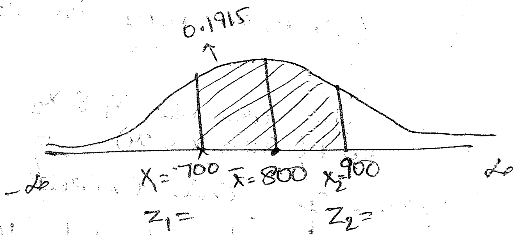
$= 73$

15) Total no of workers $= 5000$

Mean $= \bar{x} = 800$

S.D $= \sigma = 200$

a) Salary b/w 700 and 900.



$Z_1 = \dfrac{700-800}{200}$

$= \frac{1}{2} = 0.5$

$\phi(0.5) = 0.1915$

$Z_2 = \dfrac{900-800}{200} = \frac{1}{2} = 0.5$

$\phi(0.5) = 0.1915$

$\therefore$ area b/w x_1 & x_2 $= 0.1915 + 0.1915$

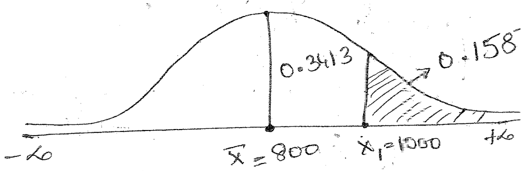
$= 0.383$

$P(700 < X < 900) = 0.383$

$\therefore$ No. of workers $= 5000 \times 0.383$

$= 1915$

b) % of workers getting salary above 1000



$Z = \dfrac{1000-800}{200}$

$= 1$

$\therefore$ from tables $\phi(1) = 0.3413$

area to the right hand side of x_1 $= 0.5 - 0.3413$

$= 0.1587$

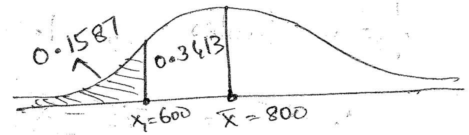
area b/w $\bar{x}$ and $x = 0.3413$.

$P(x \geq 1000) = 0.1587$

% of workers getting salary above 1000 $= 0.1587 \times 100$

$= 15.87\%$

c) % of workers getting salary below 600.



$Z = \dfrac{600-800}{200}$

$= -1$

from tables $\phi(1) = 0.3413$

area to the left hand side of x_1 $= 0.5 - 0.3413$

$= 0.1587$

$P(x \leq 600) = 0.1587$

% of workers getting salary below 600.

$= 0.1587 \times 100$

$= 15.87\%$

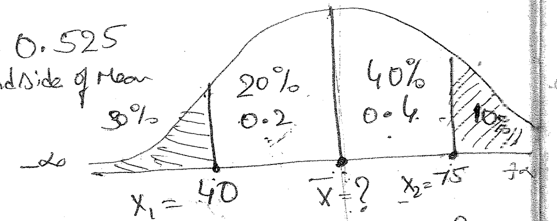
10)

area b/w X_1 & $\bar{X}$ = 0.2

for this area from tables $Z_1 = 0.525$

but Z_1 lies on the Left hand side of Mean

$\therefore Z_1 = -0.525$



$Z_1 = ?$

~~0.53~~

-0.525

$Z_2 = ?$

1.285

$0.52 = 0.1985$

0.2000

$0.53 = 0.2019$

$$Z = \frac{X - \bar{X}}{\sigma}$$

$$Z_1 = \frac{X_1 - \bar{x}}{\sigma}$$

$$-0.525 = \frac{40 - \bar{x}}{\sigma}$$

$$\bar{X} = 40 + 0.525\sigma$$

$$\boxed{\bar{X} - 0.525\sigma = 40} \quad \text{——— (1)}$$

area b/w X_2 & $\bar{X}$ = 0.4

from tables $Z_2 = 1.285$

1.28
1.29

$$1.285 = \frac{75 - \bar{x}}{\sigma}$$

$$\boxed{\bar{X} + 1.285\sigma = 75} \quad \text{——— (2)}$$

from eq (1) & (2)

subtracting (2) - (1)

$$\begin{aligned} \bar{X} + 1.285\sigma &= 75 \\ -\bar{x} + 0.525\sigma &= -40 \\ \hline 1.81\sigma &= 35 \end{aligned}$$

$$\sigma = \frac{35}{1.81} = \underline{19.337}$$

Subtituting $\sigma = 19.337$ in eq (1)

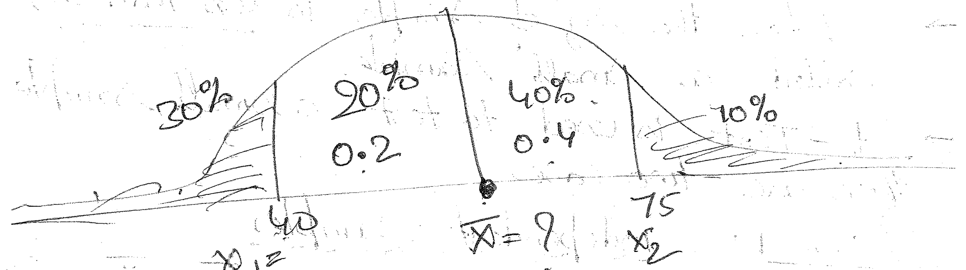
$$\bar{X} - 0.525(19.337) = 40$$

$$\bar{X} - 10.151 = 40$$

$$\bar{X} = \cancel{29.849}\ \underline{50.151}$$

$\therefore$ Mean = 50.15

S.D. = $\underline{19.34}$



$$Z = \frac{x - \bar{x}}{\sigma}$$

0.52 0.525
0.53

$$-0.525 = \frac{40 - \bar{x}}{\sigma}$$

$$\bar{x} - 0.525\sigma = 40 \quad \text{—— (1)}$$

1.28 1.28
1.29

$$1.285 = \frac{75 - \bar{x}}{\sigma}$$

$$\bar{x} + 1.285\sigma = 75 \quad \text{—— (2)}$$

31/12/52

10. Tests of Significance :

→ It is also called as tests of hypothesis

→ These tests are performed to observe the difference b/w the nature of sample and population

→ There are 5 important tests

1.) Test for Small Samples (t - Test)

2) " " Large "

~~3)~~ CHI - Square test

~~4)~~ ANOVA Test (case study compulsory)

5) F - Test (Two Samples)

Test for Small Sample (t - Test)

→ When the size of sample is less than 30, it is called a small sample.

→ t - Test is used to test a small sample

there are two cases

Case 1 : Independent Samples

* Test Statistic $= C.V = t = \dfrac{\bar{x}_1 - \bar{x}_2}{SE}\sqrt{\dfrac{n_1 n_2}{n_1+n_2}}$

Where $S.E = \sqrt{\dfrac{\Sigma dx_1^2 + \Sigma dx_2^2}{n_1+n_2-2}}$,

$= \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

* Degrees of Freedom $(d.f) = V = n_1+n_2-2$

* Level of Significance = 5% (or) 1%

  5% → 0.05, 1% → 0.01

Note : * If C.V < T.V, Null Hypothesis is accepted

If C.V > T.V, Null Hypothesis is rejected.

1) Step-1 : NULL Hypothesis (H_0)

There is no significant difference b/w the two samples

Step-2 : Test Statistic (C.V)

Given: Size of 1st sample, $n_1 = 7$

Size of 2nd sample, $n_2 = 9$

Mean of 1st Sample, $\bar{x}_1 = 196.42$

Mean of 2nd Sample, $\bar{x}_2 = 198.82$

Sum of Squares of deviation of 1st sample, $\Sigma dx_1^2 = 26.94$

" " " " of 2nd Sample, $\Sigma dx_2^2 = 18.73$

Then,

$SE = \sqrt{\dfrac{\Sigma dx_1^2 + \Sigma dx_2^2}{n_1+n_2-2}}$

$= \sqrt{\dfrac{26.94 + 18.73}{7+9-2}}$

$= 1.81$

54

$$t = \frac{\overline{x_1} - \overline{x_2}}{S.E} \cdot \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= \frac{96.42 - 198.82}{1.81} \cdot \sqrt{\frac{9 \times 7}{9+7}}$$

$C.V = 2.63$

Step 3: Table Value. (T.V)

L.O.S (Level of significance) $= 5\% = 0.05$

d.f (degrees of freedom) $= V = n_1 + n_2 - 2 = 7+9-2 = 14$

from table,
For $V = 14$, $t_{0.05} = 2.145$

Step 4: Interpretation:

$C.V = 2.63$
$T.V = 2.145$

Here $C.V > T.V$

therefore H_0 is Rejected.

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3). **Step 1:** Null Hypothesis (H_0)

There is no significant difference b/w the two batteries.

Step 2: Test Statistic (C.V)

given;

Size of 1st sample $n_1 = 10$
" of 2nd Sampe $n_2 = 12$
Mean of 1st sample $\overline{x_1} = 500$
Mean of 2nd Sample $\overline{x_2} = 560$
Variance of 1st Sample $\sigma_1^2 = 100$
Variance of 2nd Sample $\sigma_2^2 = 121$

$$S.E = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow \sqrt{\frac{100}{10} + \frac{121}{12}}$$

$$\Rightarrow \sqrt{10 + 10.083}$$

$$\Rightarrow 4.48$$

$$t = \frac{\overline{x_1} - \overline{x_2}}{S.E} \cdot \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= \frac{500 - 560}{4.48} \cdot \sqrt{\frac{10 \times 12}{10+12}}$$

$$= 13.389 \times 2.335$$

$$= 31.27$$

Table Value (T.V)

$L.O.S = 5\% = 0.05$

$d.f = V = n_1+n_2-2$

$= 10+12-2$

$= 20$

from Table

for $V = 20$, $t_{0.05} = 2.086$

Step 4 Interpretation

$C.V = 31.27$

$T.V = 2.086$

$C.V > T.V$

$\therefore$ H_0 is rejected.

4) * Null Hypothesis H_0

There is no significant difference b/w the two treatments

* Test statistics

x_1	$dx_1 = x_1 - \bar{x}_1$	dx_1^2	x_2	$dx_2 = x_2 - \bar{x}_2$	dx_2^2
14	4	16	10	-3	9
9	-1	1	15	2	4
6	-4	16	11	-2	4
10	0	0	14	1	1
11	1	1	15	2	4
$\Sigma x_1 = 50$		$\Sigma dx_1^2 = 34$	$\Sigma x_2 = 65$		$\Sigma dx_2^2 = 22$

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{50}{5} = 10$

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{65}{5} = 13$

* $SE = \sqrt{\frac{\Sigma dx_1^2 + \Sigma dx_2^2}{n_1+n_2-2}}$

$= \sqrt{\frac{34+22}{5+5-2}}$

$= 2.65$

$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} \cdot \sqrt{\frac{n_1 n_2}{n_1+n_2}}$

$= \frac{10-13}{2.65} \cdot \sqrt{\frac{5 \times 5}{5+5}}$

$= 1.79$

* Table Value (T.V)

$LOS = 5\% = 0.05$

$df = V = n_1+n_2-2 = 5+5-2 = 8$

From table,

For $V = 8$, $t_{0.05} = 2.306$

* Interpretation:

$C.V = 1.79$

$T.V = 2.306$

~~C.V > T.~~

$C.V < T.V$

$\therefore$ H_0 is accepted.

58 Test for small samples t - Test

* Case II : dependent samples (Before - After)

$$\boxed{t = \frac{\bar{d}\sqrt{n}}{S.E}}$$

$\bar{d} = \frac{\Sigma d}{n}$

d = After − Before

* $S.E = \sqrt{\frac{\Sigma d^2 - n(\bar{d})^2}{n-1}}$

1)

Before	68	75	60	55	40	80	65	72	64	42
After	65	78	62	53	45	75	62	68	60	45

Sol: step 1: Null Hypothesis H_0

There is no significant difference in weight control befor & after Yoga programme i.e., Yoga is not useful

Step 2: Test statistic (c.v)

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Before	After	d = After − Before	d^2
68	65	−3	9
75	78	3	9
60	62	2	4
55	53	−2	4
40	45	5	25
80	75	−5	25
65	62	−3	9
72	68	−4	16
64	60	−4	16
42	45	3	9
		$\Sigma d = -8$	$\Sigma d^2 = 126$

* $\bar{d} = \frac{\Sigma d}{n} = \frac{-8}{10} = -0.8$

* $S.E = \sqrt{\frac{\Sigma d^2 - n(\bar{d})^2}{n-1}} = \sqrt{\frac{126 - 10(-0.8)^2}{10-1}}$

$= \sqrt{\frac{126 - 6.4}{9}}$

$= 3.645$

* $t = \left|\frac{\bar{d}\sqrt{n}}{S.E}\right|$

$= \frac{|-0.8 \cdot \sqrt{10}|}{3.645} = 0.694$

.6135

* ~~Table Value (T.V)~~

* Interpretation.

Level of significance

LOS = 5% = 0.05

d.f = $V = n-1 \Rightarrow 10-1 = 9$

from Tables,

for $V = 9$, $t_{0.05} = 2.262$

C.V = 0.694

T.V = 2.262

$\therefore$ C.V < T.V

$\therefore$ H_0 is accepted

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TEST for Large Samples.

* When the size of a sample is 30 or more, The sample is called Large sample.

* Test statistic (C.V) = $\dfrac{|\bar{X}_1 - \bar{X}_2|}{S.E}$

* $S.E = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Table Value:

$\boxed{\begin{aligned} 1\% &= 2.58 \\ 5\% &= 1.96 \end{aligned}}$

1) * Null Hypothesis H_0

There is no significant difference b/w the two samples.

* Calculation of test statistics:

Size of first sample = $n_1 = 121$

" " Second sample = $n_2 = 81$

Mean of first sample = $\bar{X}_1 = 84$

" " Second sample = $\bar{X}_2 = 81$

S.D of first sample $\sigma_1 = 10$

" " Second sample $\sigma_2 = 12$

* $S.E = \sqrt{\dfrac{(10)^2}{121} + \dfrac{(12)^2}{81}} = \sqrt{0.8264 + 1.777}$

$= 1.6135$

$C.V = \frac{|\bar{x}_1 - \bar{x}_2|}{S.E}$

* $C.V = \frac{|84 - 81|}{1.613} = \frac{3}{1.613}$

$= \underline{\underline{1.859}}$

Interpretation:

Level of significance LOS $= 5\% = 0.05$

table value at 5% of LOS $= 1.96$

but $C.V = 1.859$

$\therefore$ $C.V < T.V$

$\therefore$ null hypothesis is accepted.

CHI – SQUARE TEST (χ^2 – Test)

* It is one of the simplest & most widely used non-parametric test
* CHI-square Test is generally used where the data is divided into two broad categories and each category may be divided into sub-categories
* Test statistic $= \boxed{C.V = \chi^2 = \Sigma \left[\frac{(O-E)^2}{E}\right]}$

where O = Observed values or actual data

E = expected values

$E = \frac{RT \times CT}{N}$

RT = Row total

CT = Column Total.

N = Grand Total.

degrees of freedom $d.f = V = (r-1)(c-1)$

(or)

$V = n-1$

1) Null hypothesis H_0

There is no significant difference in Tea consuming habbit b/w the two religions.

Observed Values: (O)

	Tea	non-Tea	Total
Hindu			
Non Hindu			
Total.			

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F – Test

* When two samples are taken from two different populations of the same nature,
* F-Test will be used.

Test statistic (C.V) = $F = \dfrac{S_1^2}{S_2^2}$

where S_1^2 = Variance of the Larger Sample
S_2^2 = Variance of the Smaller Sample

$$S_1^2 = \frac{\Sigma dx_1^2}{n_1 - 1}$$

$$S_2^2 = \frac{\Sigma dx_2^2}{n_2 - 1}$$

degrees of freedom df. $V_1 = n_1 - 1$
$V_2 = n_2 - 1$

1) Sample 1 : 20 76 26 27 23 22 18 24 25 19 20
Sample 1 : 17 23 32 25 22 24 28 18 31 33 27

2) 62 Sample I : 60 65 71 74 76 82 85 87 – –
Sample II : 61 66 67 85 78 63 85 86 88.9

* Null hypothesis H_0:
There is no significant difference b/w the two samples

* Calculation of Test statistics:

Larger sample x_1	$dx_1 = x_1 - \bar{x}_1$	dx_1^2	Smaller sample x_2	$dx_2 = x_2 - \bar{x}_2$	dx_2^2
61	−16	256	60	−15	225
66	−11	121	65	−10	100
67	−10	100	71	−4	16
85	8	64	74	−1	1
78	1	1	76	1	1
63	14	196	82	7	49
85	8	64	85	10	100
86	9	81	87	12	144
88	11	121			
91	14	196			
$\Sigma x_1 = 770$		$\Sigma dx_1^2 = 1200$	$\Sigma x_2 = 600$		$\Sigma dx_2^2 = 636$

* $\bar{x}_1 = \dfrac{\Sigma x_1}{n_1} = \dfrac{770}{10} = 77$

* $\bar{x}_2 = \dfrac{\Sigma x_2}{n_2} = \dfrac{600}{80} = \cancel{60}\ 75$

* $s_1^2 = \dfrac{\Sigma dx_1^2}{n_1 - 1} = \dfrac{1200}{9}$
133.33

* $S_2^2 = \dfrac{\Sigma dx_2^2}{n_2 - 1} = \dfrac{636}{7}$
$= 90.86$

* $F = \frac{S_1^2}{S_2^2} = \frac{183.33}{90.86}$

$= 1.47$

Table Value:

LOS = 5% = 0.05

d.f $V_1 = n_1 - 1 = 9$ }M

$V_2 = n_2 - 1 = 7$ }N

For $V_1 = 9$, $V_2 = 7$, $F_{0.05} = 3.6767$

Interpretation.

CV = 1.47

T.V = 3.6767

Here C.V < T.V

∴ H_0 is accepted

ANOVA TEST

It is also a kind of F Test.

* When 3 or More Samples are taken from
* 3 or More different populations of the Same nature Anova Test one way classification is used.
* ANOVA means analysis of Variants.
* ~~Test~~

Steps in the procedure:

Step 1) : Null hypothesis H_0

Step 2 : Calculation of Sample Means and Combined Mean.

Step 3 : Sum of squares b/w Samples

Step 4 : Sum of Squares with in samples

Step 5 : ANOVA Table.

Step 6 : Test Statistic

Step 7 : Table Value.

Step 8 : Interpretation.

1) Null hypothesis H_0

There is no Significant difference among the 3 Samples

2) Calculation of Sample Mean and Combined mean

Mean of Sample A $= \bar{X}_1 = \dfrac{14+16+18}{3} = 16$

" B $= \bar{X}_2 = \dfrac{14+13+15+22}{4} = 16$

" C $= \bar{X}_3 = \dfrac{18+16+17+19+20}{5} = 18$

Combined Mean $= \bar{X} = \dfrac{\bar{X}_1+\bar{X}_2+\bar{X}_3}{3} = \dfrac{16+16+18}{3}$

$= 16.67$

Sum of squares b/w Samples:

$= n_1(\bar{x}_1-\bar{x})^2 + n_2(\bar{x}_2-\bar{x})^2 + n_3(\bar{x}_3-\bar{x})^2$

$3(16-16.67)^2 + 4(16-16.67)^2 + 5(18-16.67)^2$

$3(0.4489) + 4(0.4489)^2 + 5(1.7689)$

$1.3467 + \overset{1.7956}{\cancel{5.3868}} + 8.8445$

$= 11.9868 \; \cancel{15.578}$

Sum of Squares with in Samples:

X_1	$(x_1-\bar{x}_1)^2$	x_2	$(x_2-\bar{x}_2)^2$	x_3	$(x_3-\bar{x}_3)^2$
14	$(14-16)^2=4$	14	$(14-16)^2=4$	18	$(18-18)^2=0$
16	$(16-16)^2=0$	13	$(13-16)^2=9$	16	$(16-18)^2=4$
18	$(18-16)^2=4$	15	$(15-16)^2=1$	17	$(17-18)^2=1$
		22	$(22-16)^2=36$	19	$(19-18)^2=1$
				20	$(20-18)^2=4$
	8		50		10

$\therefore$ Sum of Squares with in Samples $= 8+50+10 = 68$

Step5 Anova table

Source	Sum of Squares	d.f	Mean Square
B/w Samples	11.9868	V_1 = no of samples − 1 = 3 − 1 = 2	$\frac{11.9868}{2} = 5.9934$
Within Samples	68	V_2 = no of observations − No of samples, 12 − 3 = 9	$\frac{68}{9} = 7.55$

Step6: Test statistics

f = Mean Square with in Samples (or) Larger Mean Square Value by smaller Mean Square Value

$f = \dfrac{7.55}{5.99} = 1.2604$

Step7 T.V

for $V_1 = 2$, $V_2 = 9$ & $F_{0.05} = 4.2565$ of table.

Step8: Interpretation.

C.V = 1.2604

T.V = 4.2565

CV < TV

$\therefore H_0$ is accepted

Anova Test two way Classification

steps

1) Two Null hypothesis
2) ~~Null hypothesis~~ Simplification of data
3) Correction factor
4) Finding sum of squares b/w rows.
5) Finding Sum of squares b/w Columns
6) Finding Total sum of squares
7) Anova Table
8) Caliculation of two test statistics.
9) Finding Two table Values
10) Making two interpretations.

Q) 50

Water Temperature	Detergent powder A	B	C
Cold	57	55	64
Warm	49	52	68
Hot	54	46	58

Perform a two way analysis of Variance using 5% Level of Significance.

Step 1: Null hypothesis.

1) There is no Significant, difference among the Water temperatures.
2) There is no Significant difference among the Quality of the detergent powders.

Step 2: Simplification of data

Let us subtract 50 from all the nine elements to simplify the data then we get

Water temperature	Detergent powder A	B	C	Total
Cold	7	5	17	29
Warm	-1	2	18	19
Hot	4	-4	8	8
Total	10	3	43	56

Step 3: Correction factor:

$$C.F = \frac{T^2}{N} = \frac{(56)^2}{9} = 348.44$$

Step 4: Sum of squares b/w rows: − CF

$$= \left(\frac{29^2}{3} + \frac{19^2}{3} + \frac{8^2}{3}\right) - C.F$$

$$= 422 - 348.44$$

$$= 73.56$$

Step 5: Sum of squares b/w Columns.

$$\frac{10^2}{3} + \frac{3^2}{3} + \frac{43^2}{3} - CF$$

$$= 304.22$$

Step 6: Total sum of squares.

$$(49 + 25 + 289 + 144 + 324 + 16 + 16 + 64) - CF$$

$$788 - 348.44$$

$$439.56$$

$439.56 - 377.78 = 61.78$

Step 7: Anova table:

Source	Sum of Squares	Degrees of Freedom (df)	Mean square
B/w Rows (Water Temp)	73.56	V_1 = No. of Rows − 1 = 3 − 1 = 2	$\frac{73.56}{2} = 36.78$
B/w Columns (Det powder)	304.22	V_1 = No. of columns − 1 = 3 − 1 = 2	$\frac{304.22}{2} = 152.11$
Residual (Bal. fig.)	61.78	V_2 = 4	$\frac{61.78}{4} = 15.44$
Total:	439.56	V = no. of observations − 1 = 9 − 1 = 8	

Residual = Total − (sum of squares b/w Rows + Columns)

7° Step 8: Calculation of two test statistics:

1) for Rows: $F = \frac{\text{Mean square b/w rows}}{\text{Mean square of Residual}}$

$$= \frac{36.78}{15.44} = 2.38$$

2) for Columns $F = \frac{\text{Mean Square b/w Columns}}{\text{Mean Square of Residual}}$

$$= \frac{152.11}{15.44} = 9.85$$

Step 9: Finding Two table values.

for Rows: for $V_1 = 2$ $V_2 = 4$ $F_{0.05} = 6.9443$

for columns for $V_1 = 2$ $V_2 = 4$ $F_{0.05} = 6.9443$

Step 10: Interpretation:

for Rows: (Water temp.)

C.V = 2.38

T.V = 6.94

C.V < T.V H_0 is accepted

for Columns (Det powder)

C.V = 9.85

T.V = 6.94

CV > TV H_0 is Rejected.

∴ The data is influenced by the detergent powders but not with the water temp.